### Domain One: Number Sense

<table>
<thead>
<tr>
<th>1.1Numbers, Relationships Among Numbers, and Number Systems</th>
<th>1.2Computational Tools, Procedures, and Strategies.</th>
</tr>
</thead>
<tbody>
<tr>
<td>• understand base ten place value, number theory concepts (e.g., greatest common factor), and the structure of the whole, integer, rational, and real number systems.</td>
<td>• demonstrate fluency in standard algorithms for computation and evaluate the correctness of nonstandard algorithms.</td>
</tr>
<tr>
<td>• order integers, mixed numbers, rational numbers (including fractions, decimals, and percents) and real numbers.</td>
<td>• demonstrate an understanding of the order of operations.</td>
</tr>
<tr>
<td>• represent numbers in exponential and scientific notation.</td>
<td>• round numbers, estimate the results of calculations, and place numbers accurately on a number line.</td>
</tr>
<tr>
<td>• describe the relationships between the algorithms for addition, subtraction, multiplication, and division.</td>
<td>• demonstrate the ability to use technology, such as calculators or software, for complex calculations.</td>
</tr>
<tr>
<td>• understand properties of number systems and their relationship to the algorithms, [e.g., 1 is the multiplicative identity; (27 + 34 = 2 \times 10 + 7 + 3 \times 10 + 4 = (2 + 3) \times 10 + (7 + 4))].</td>
<td></td>
</tr>
<tr>
<td>• perform operations with positive, negative, and fractional exponents, as they apply to whole numbers and fractions.</td>
<td></td>
</tr>
</tbody>
</table>

### Domain Four: Statistics, Data Analysis, and Probability

<table>
<thead>
<tr>
<th>4.1Collection, Organization, and Representation of Data</th>
<th>4.2Inferences, Predictions, and Arguments Based on Data.</th>
<th>4.3Basic Notions of Chance and Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>• represent a collection of data through graphs, tables, or charts.</td>
<td>• interpret a graph, table, or chart representing a data set.</td>
<td>• define the concept of probability in terms of a sample space of equally likely outcomes.</td>
</tr>
<tr>
<td>• understand the mean, median, mode, and range of a collection of data.</td>
<td>• draw conclusions about a population from a random sample, and identify potential sources and effects of bias.</td>
<td>• use their understanding of complementary, mutually exclusive, dependent, and independent events to calculate probabilities of simple events</td>
</tr>
<tr>
<td>• have a basic understanding of the design of surveys, such as the role of a random sample.</td>
<td></td>
<td>• can express probabilities in a variety of ways, including ratios, proportions, decimals, and percents.</td>
</tr>
</tbody>
</table>
Domain Two: Algebra and Functions

2.1 Patterns and Functional Relationships.
- represent patterns, including relations and functions, through tables, graphs, verbal rules, or symbolic rules.
- use proportional reasoning such as ratios, equivalent fractions, and similar triangles, to solve numerical, algebraic, and geometric problems

2.2 Linear and Quadratic Equations and Inequalities.
- find equivalent expressions for equalities and inequalities, explain the meaning of symbolic expressions (e.g., relating an expression to a situation and vice versa).
- find the solutions, and represent them on graphs.
- recognize and create equivalent algebraic expressions (e.g., 2(a+3) = 2a + 6).
- represent geometric problems algebraically (e.g., the area of a triangle).
- have a basic understanding of linear equations and their properties (e.g., slope, perpendicularity); the multiplication, division, and factoring of polynomials; and graphing and solving quadratic equations through factoring and completing the square.
- interpret graphs of linear and quadratic equations and inequalities, including solutions to systems of equations.

Domain Three: Measurement and Geometry

3.1 Two- and Three-dimensional Geometric Objects
- understand characteristics of common two- and three-dimensional figures, such as triangles (e.g., isosceles and right triangles), quadrilaterals, and spheres.
- draw conclusions based on the congruence, similarity, or lack thereof, of two figures.
- identify different forms of symmetry, translations, rotations, and reflections.
- understand the Pythagorean theorem and its converse.
- are able to work with properties of parallel lines.

3.2 Representational Systems, Including Concrete Models, Drawings, and Coordinate Geometry
- use concrete representations, such as manipulatives, drawings, and coordinate geometry to represent geometric objects.
- construct basic geometric figures using a compass and straightedge.
- represent three-dimensional objects through two-dimensional drawings.
- combine and dissect two- and three-dimensional figures into familiar shapes, such as dissecting a parallelogram and rearranging the pieces to form a rectangle of equal area.

3.3 Techniques, Tools, and Formulas for Determining Measurements
- estimate and measure time, length, angles, perimeter, area, surface area, volume, weight/mass, and temperature through appropriate units and scales.
- identify relationships between different measures within the metric or customary systems of measurements and estimate an equivalent measurement across the two systems.
- calculate perimeters and areas of two-dimensional objects and surface areas and volumes of three-dimensional objects.
- relate proportional reasoning to the construction of scale drawings or models.
- use measures such as miles per hour to analyze and solve problems.
Stupid Math Tricks

Scoring well on ten stupid math questions will ensure that you’ll be close to reaching the sixteen questions required on the exam.

1. Don’t miss the plug-n-play questions.

2. Don’t miss factoring questions.

3. Select the hard math questions carefully.
## Domain One: Number Sense

### 1.1 Numbers, Relationships Among Numbers, and Number Systems

1A understand base ten place value, number theory concepts (e.g., greatest common factor), and the structure of the whole, integer, rational, and real number systems.

#### 6. Simplify the expression below.

\[
\frac{5}{8} + \frac{3}{4} - \frac{7}{3} - \frac{3}{5}
\]

A. \(\frac{1}{2}\)
B. \(2\frac{1}{2}\)
C. 4
D. 5

ANS D
D. 5

#### Simplify the equation below:

\[
\frac{\sqrt[3]{x^{-4}} \cdot \sqrt[4]{y^3}}{\sqrt[3]{x} \cdot \sqrt[4]{y^3}}
\]

A. \(x^{-1} y^{\frac{1}{2}}\)
B. \(xy^{-\frac{3}{4}}\)
C. \(x^{-\frac{9}{4}} y\)
D. \(xy^{-\frac{3}{4}}\)

ANS D
D. \(xy^{-\frac{3}{4}}\)

#### If \(\frac{a + 13b}{3a - b} = 3\), what is the value of \(\frac{a^3}{b^3}\)?

A. \((-\frac{8}{5})^3\)
B. \((-\frac{7}{4})^3\)
C. \(2^3\)
D. \(7^3\)

ANS C
c. \(2^3\)
7. If the number 504 is written as a product of its prime factors in the form $a^3b^2c$, which of the following is the numerical value of $a + b + c$?

A. 5  
B. 6  
C. 12  
D. 15

1B order integers, mixed numbers, rational numbers (including fractions, decimals, and percents) and real numbers.

1C represent numbers in exponential and scientific notation.

Use the sequence of steps below to answer the question that follows.

$4.26 \times 2.2 = \left(426 \times \frac{1}{100}\right) \times \left(22 \times \frac{1}{10}\right)$

$= 426 \times \left(\frac{1}{100} \times 22\right) \times \frac{1}{10}$

$= 426 \times \left(22 \times \frac{1}{100}\right) \times \frac{1}{10}$

$= (426 \times 22) \times \left(\frac{1}{100} \times \frac{1}{10}\right)$

$= (426 \times 22) \times \frac{1}{1000}$

The sequence of steps above could be used to answer which of the following questions?

A. How can a decimal be converted to a fraction reduced to lowest terms?
B. Where should the decimal point be placed in the product of two decimals?
C. Why is it necessary to invert and multiply when solving problems involving fractions?
D. How is scientific notation used to multiply decimals?
10. A scientist observes that the population of bacteria cells in a culture doubles every 5 hours. If the initial population is 10,000 cells, which of the following equations could be used to predict the number of cells at any time $t$?

A. $(10,000)2^{0.2t}$  
B. $(10,000)2^{5t}$  
C. $(10,000)2^{0.5t}$  
D. $(10,000)5^{2t}$

A scientist determines that the amount of mold growing in a laboratory culture every four days. If there is originally 0.5 grams of mold in the culture, which of the following equations best models the amount of mold, $M$, at a given time, $t$?

A. $M = 0.5(3)^{0.25t}$  
B. $M = 0.5(3)^{4t}$  
C. $M = 3t + 0.5$  
D. $M = t^3 + 0.5$

2. Use the information below to answer the question that follows.

The Sahara Desert covers about $8.3 \times 10^{13}$ square feet. The average depth of the sand in the Sahara Desert is 200 feet. A grain of sand has a volume of approximately $1.3 \times 10^{-9}$ cubic feet.

Which of the following is the best estimate of the number of grains of sand in the Sahara Desert?

A. $10^{22}$  
B. $10^{23}$  
C. $10^{24}$  
D. $10^{25}$
describe the relationships between the algorithms for addition, subtraction, multiplication, and division.

4. A sweater that usually sells for $39.98 is marked down 20% for an end-of-the-season clearance sale. If the sales tax is 6%, how much would a customer pay for the sweater at the sale price?

A. $ 8.00  
B. $ 31.98  
C. $ 33.90  
D. $ 34.38  

Correct Response: C. The first step in solving this problem is to determine the sale price of the sweater. Since the sweater has been marked down 20%, it is now selling for 80% of its original price. $(0.80)(39.98) = 31.98$

Next, add the sales tax by multiplying the sale price by 1.06 (representing 106%). $(1.06)(31.98) = 33.90$

Therefore, a customer would pay $33.90 for the sweater at the sale price. Choice A represents only the amount of the discount, not the final price of the sweater. Choice B represents the sale price of the sweater before sales tax has been added. Choice D represents the amount the sweater would have cost at a 20% discount instead of a 20% discount with a 6% tax added.

12. What is 4% of 400?

A. 16  
B. 64  
C. 100  
D. 160  

Correct Response: A. Four percent (4%) of something is the same as four hundredths of something. To determine four hundredths of 400, you would multiply 0.04 by 400. The result is 16.
2. Which of the following situations represents \( \frac{2\frac{2}{5}}{1\frac{1}{4}} \)?

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A ( 2\frac{2}{5} ) acre lot needs to be plowed. If four workers split the plowing evenly, how many acres will each person plow?</td>
</tr>
<tr>
<td>B</td>
<td>One side of a one-fourth-square-foot rectangle is ( 2\frac{2}{5} ) feet. How long is the other side?</td>
</tr>
<tr>
<td>C</td>
<td>Alix sawed off one fourth of a ( 2\frac{2}{5} ) yard log. How many yards did Alix saw off?</td>
</tr>
<tr>
<td>D</td>
<td>Terry picked ( 2\frac{2}{5} ) pounds of berries and put them into one quarter-pound containers. How many containers of berries did Terry fill?</td>
</tr>
</tbody>
</table>

6. Use the word problem below to answer the question that follows.

Janelle calls her friend Frank long-distance. The first minute of the call costs $1.23, and each additional minute costs $0.89. The total cost of the call is $15.47. For how many minutes did Janelle and Frank talk?

If \( x \) represents the total number of minutes talked, which of the following equations can be used to solve this problem?

<table>
<thead>
<tr>
<th>Option</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( 1.23 + 0.89 = 15.47 )</td>
</tr>
<tr>
<td>B</td>
<td>( 1.23 + 0.89(x - 1) = 15.47 )</td>
</tr>
<tr>
<td>C</td>
<td>( (1.23 + 0.89)x = 15.47 )</td>
</tr>
<tr>
<td>D</td>
<td>( (1.23 + 0.89)(x - 1) = 15.47 )</td>
</tr>
</tbody>
</table>

22. During a bike-a-thon, a local company pledges to donate $1.25 for every $4.00 pledged by the public. If the public pledges a total of $156.00 dollars per mile, how much will the company donate permile?

<table>
<thead>
<tr>
<th>Option</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$2.75</td>
</tr>
<tr>
<td>B</td>
<td>$48.75</td>
</tr>
<tr>
<td>C</td>
<td>$195.00</td>
</tr>
<tr>
<td>D</td>
<td>$499.20</td>
</tr>
</tbody>
</table>
The nutritional label on a snack food indicates that one serving of the snack provides 20% of the daily adult requirement of protein. If one serving weighs $1 \frac{3}{4}$ ounces, how many ounces of the snack food would provide 100% of the daily requirement of protein?

A. 3 ounces

B. $8 \frac{3}{4}$ ounces

C. 11.43 ounces

D. $21 \frac{3}{4}$ ounces

1E understand properties of number systems and their relationship to the algorithms, [e.g., 1 is the multiplicative identity; $27 + 34 = 2 \times 10 + 7 + 3 \times 10 + 4 = (2 + 3) \times 10 + (7 + 4)].

10. Use the diagram below to answer the question that follows.

The diagram above could be used to model which of the following?

A. a function machine
B. the associative property
C. prime factorization
D. conditional probability

Correct Response: B. The associative property for addition states that when adding numbers the grouping of the addends can be changed and the sum remains the same. In the diagram $(4 + 5) + 2 = 4 + (5 + 2) = 11$. Thus the diagram models the associative property.

Use the passage below to answer the question that follows.

Think of two tasks and determine whether or not the order in which you do them matters. For example, does it matter whether you wash your face first or brush your teeth first? Does it matter whether you brush your teeth first or eat your chocolate cookie first?

This passage would be most helpful in gaining a deeper understanding of which of the following mathematical concepts?

A. commutative versus noncommutative systems
B. cardinal numbers versus ordinal numbers
C. congruence versus similarity
D. equations versus functions
<table>
<thead>
<tr>
<th>48</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>× 17</td>
<td>× 17</td>
</tr>
<tr>
<td>336</td>
<td>56</td>
</tr>
<tr>
<td>480</td>
<td>280</td>
</tr>
<tr>
<td>816</td>
<td>80</td>
</tr>
<tr>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>816</td>
<td>816</td>
</tr>
</tbody>
</table>

| standard multiplication algorithm | expanded multiplication algorithm |

A significant difference between the standard multiplication algorithm and the expanded multiplication algorithm is that the expanded multiplication algorithm:

A. can be used with both positive and negative integers.
B. **emphasizes the use of the distributive property.**
C. minimizes the need to memorize multiplication facts.
D. makes knowledge of place value unnecessary.

27. If $6b + 20 = a$, and $4b + 30 = a$, then $b =$
   - A. -5
   - B. -1
   - C. 1
   - D. 5

30. A theorem is expressed in the form below.
   If $A$ then $B$. How can both the above theorem and its converse be expressed?
   - A. If $B$ then $A$.
   - B. If not $B$ then not $A$.
   - C. If not $A$ then not $B$.
   - D. **$A$ if and only if $B$.**
Third-grade students are asked to figure out three different ways to calculate the perimeter of an 8 × 7 rectangle. The students develop the following three methods, and note that they yield the same result.

• add each side (8 + 7 + 8 + 7)
• multiply the length and width by two, and add the products (2 × 8) + (2 × 7)
• add the length and width and multiply the sum by two 2(8 + 7)

This activity is most likely to promote students' understanding of:

A. number properties such as the distributive property.
B. congruence relationships for quadrilaterals.
C. translating between numeric and symbolic expressions.
D. prime and composite factors.

Use the information below to answer the question that follows.

| 12 |
| 29 |
| 88 |
| +11 |

When given the addition problem above, a student quickly said "140." When asked how she solved the problem, the student replied, "I added 88 and 12 to get 100, and 29 and 11 to get 40. Then I added these two numbers together." Which two number properties of addition did the student use in solving this problem?

A. associative and commutative
B. associative and additive identity
C. commutative and additive identity
D. distributive and additive inverse

putational algorithms
1F perform operations with positive, negative, and fractional exponents, as they apply to whole numbers and fractions.

3. Simplify the expression below.

\[
\frac{5 + 3}{8 - 4} + \frac{7 - 3}{8 - 5}
\]

A. \(\frac{1}{2}\)

B. \(2\frac{1}{2}\)

C. 4

D. 5

1.2 Computational Tools, Procedures, and Strategies.

1G demonstrate fluency in standard algorithms for computation and evaluate the correctness of nonstandard algorithms.

A student is having difficulty with multiplication problems that involve a two-digit number times a one-digit number. Three examples of the student's work are shown below:

A. \[27 \times 4 = 108\]

B. \[34 \times 6 = 204\]

C. \[43 \times 5 = 215\]

D. \[52 \times 7 = 364\]

A. \[27 \times 4 = 108\]

B. \[34 \times 6 = 204\]

C. \[43 \times 5 = 215\]

D. \[52 \times 7 = 364\]

The answer is A.
12. Crystal estimates that she spends 13% of each paycheck on rent and 14% of each paycheck on food. If her paycheck is $300.00, how much does she have left over after paying for rent and food?

A. $125.00  
B. $175.00  
C. $214.29  
D. $275.00

10. Select the number that is missing in the problem below.
1,806 – ______ = 358
A. 1,448  
B. 1,756  
C. 2,164  
D. 2,774

Correct Response: A. This problem asks you to find the number that, when subtracted from 1,806, equals 358. That number is the same as 1,806 minus 358, which is 1,448.

10. How many five-letter combinations can be formed from the word teaching?
A. \( \frac{8!}{5!} \)  
B. \( \frac{8!}{5!3!} \)  
C. \( \frac{5!}{3!} \)  
D. \( 5! \)

The correct response is B. A general formula for a combination of \( n \) elements taken \( r \) at a time is:

\[
\binom{n}{r} = \frac{n!}{r!(n-r)!}
\]

In this problem, the eight elements (letters) in the word teaching are to be taken five at a time. Therefore the solution is set up as shown below:

\[
8^c5 = \frac{8!}{5!(8-5)!} = \frac{8!}{5!3!}
\]
7. **Read the word problem below; then answer the question that follows.**

In order to get an A in her math class, Alma must have an average of at least 90 for her test scores. Alma's scores on the first two tests were 85 and 92. To get an A in math, what scores can Alma get on her third test?

If \( y \) represents Alma's score on the third test, which of the following expresses the inequality for the word problem stated above?

- A. \( 3(85 + 92) > 90y \)
- B. \( \frac{85 + 92 + y}{3} > 90 \)
- C. \( 85 + 92 + y > 90 \div 3 \)
- D. \( \frac{3y}{85 + 92} > 90 \)

The correct answer is B.

B. \( \frac{85 + 92 + y}{3} > 90 \)
Use the two numbers below to answer the question that follows.

\[ N = 6 \times 10^2 + 4 \times 10^1 + 7 \times 10^0 + 3 \times 10^{-1} \]
\[ M = 1 \times 10^2 + 4 \times 10^1 + 9 \times 10^{-1} \]

The numbers \( N \) and \( M \) are given above. If the product of \( N \) and \( M \) is expressed in expanded notation, what is the value of \( n \) in the term \( n \times 10^2 \)?

A. 3  
B. 6  
C. 7  
D. 9

What is the reciprocal of the mixed number \( 5 \frac{3}{b} \) where \( b \) is a whole number?

A. \( \frac{5b}{8} \)  
B. \( \frac{b}{8} \)  
C. \( \frac{5b}{5b + 3} \)  
D. \( \frac{b}{5b + 3} \)

The correct answer is D.

Keisha spent ______ minutes doing tricks on her yo-yo. She spent the first ______ minutes doing the "forward pass." She spent the next ______ minutes doing "around the world." Then she stopped.

If placed in the blanks in the order shown, which set of numbers is a correct solution to the problem above?

A. 20, 5, 10  
B. 5, 10, 15  
C. 25, 10, 15  
D. 10, 5, 20
Several groups have participated in a community food drive to collect canned food. Group A brought in \( \frac{4}{9} \) of all the cans collected by the community. Group B brought in \( \frac{1}{3} \) of all the cans collected by the community.

How much of the community’s total did these two groups combined collect?

\[
\frac{4}{9} + \frac{1}{3}
\]

An individual has begun to solve the problem, as shown above. Which of the following steps should the individual take next to determine the correct answer?

A. Find the least common multiple of the numerators.
B. Invert the numerator and denominator in the smaller fraction.
C. Reduce both of the fractions to their lowest terms.
D. Find the lowest common denominator of the fractions.

Dana and Tyler had a party at which they served pizza, fruit punch, crackers with dip, and egg rolls.

Unfortunately, one of these foods caused several guests to develop food poisoning. At the party, Larry ate pizza and crackers with dip and became ill the next day. Matsako ate each of the foods at the party and she also became ill. Torry had fruit punch and pizza and he did not become ill. Lyle and Carrie had fruit punch, crackers with dip, and egg rolls, and they became ill. Annette had pizza and egg rolls and she did not become ill. Which food most likely caused the food poisoning?

Which of the following problem-solving strategies would be most appropriate to use to solve this problem?

A. Work backwards.
B. Make comparative lists.
C. Set up an equation.
D. Use a manipulative.

Correct Response: B. The most efficient way of solving this problem is to begin by making one list of all the people who became ill and the foods each of them ate during the party. Then make another list of all of the people who did not become ill and the foods that each of those people ate. Next, cross off foods that appear on both lists. The foods that remain on the list of people who became ill are the ones most likely to have caused the illness.
5. Students in a fifth-grade math class are given the following assignment.

Write the following problem in your math journal: $8.7 \times 3.28$.

Show how you would estimate the product using the digits to the left of the decimal point. Then show how you would estimate the product by rounding.

Which method produces an estimate that is closer to the right answer? Explain how you know.

This assignment is likely to be most effective in encouraging students to:

A. use order of operations.
B. apply and evaluate mental math strategies.
C. work backward to solve problems.
D. recognize and compare properties of numbers.

Use the word problem below to answer the question that follows.

Janelle calls her friend Frank long-distance. The first minute of the call costs $1.23, and each additional minute costs $0.89. The total cost of the call is $15.47. For how many minutes did Janelle and Frank talk?

If $x$ represents the total number of minutes talked, which of the following equations can be used to solve this problem?

A. $1.23 + 0.89 = 15.47$
B. $1.23 + 0.89(x - 1) = 15.47$
C. $(1.23 + 0.89)x = 15.47$
D. $(1.23 + 0.89)(x - 1) = 15.47$
Step | Statement | Expression
--- | --- | ---
1 | Pick any number. | \( n \)
2 | Multiply it by three. | \( 3n \)
3 | Add six to the result. | \( 3n + 6 \)
4 | Divide by three. | ---
5 | Subtract the number picked first. | 2

The table above shows an incomplete algebraic proof of a number trick that always results in the number 2, regardless of which number is initially picked. Which of the following expressions should appear in Step 4 to complete the proof?

A. \( n + 2 \)
B. \( n + 6 \)
C. \( 3n + 2 \)
D. \( 3n + 3 \)

The student council is able to purchase pencils with the school name printed on them for $9.80 per box of pencils. To raise money for a winter carnival, the student council plans to sell the pencils for $0.25 each. How much profit can be made on each box of pencils?

Which of the following pieces of information is essential for answering the question?

A. number of boxes of pencils sold
B. number of people who bought pencils
C. number of pencils in each box
D. number of pencils sold

Correct Response: C.

The profit made on each box of pencils may be determined by multiplying the number of pencils in each box times $0.25 per pencil and then subtracting the cost of the box of pencils ($9.80). The number of pencils per box cannot be determined from the information stated in the problem. Therefore, choice C is the correct response. The information described in choices A, B, and D is not required to determine the profit that can be made on each box of pencils.
11. Which number is a multiple of 7?
A. 27
B. 48
C. 52
D. 63

Correct Response: D. The multiples of a number result from multiplying that number by the whole numbers 0, 1, 2, 3, and so on. The first eleven multiples of 7 are 0, 7, 14, 21, 28, 35, 42, 49, 56, 63, and 70. Thus, only Response D is a multiple of 7.

1J demonstrate the ability to use technology, such as calculators or software, for complex calculations.

9. 0.07 is equal to:
A. \( \frac{7}{100} \)
B. 17
C. \( \frac{7}{10} \)
D. \( \frac{10}{7} \)

Correct Response: A. Numbers to the right of a decimal point indicate a fraction of a whole. The first number to the right of a decimal point indicates tenths, the second number to the right of a decimal point indicates hundredths, the third number to the right of a decimal point indicates thousandths, and so on. Thus, the number 0.07 is the same as seven hundredths or the fraction \( \frac{7}{100} \).

On a calculator, a student starts with 2 and takes the square root four times in succession. This is equivalent to calculating which of the following numbers?
A. \( 2^{0.25} \)
B. \( (\sqrt{2})^\frac{1}{3} \)
C. \( (\sqrt{2})^\frac{1}{4} \)
D. \( \frac{1}{2^{16}} \)

ANS D
D. \( \frac{1}{2^{16}} \)
29. A shopper sees some gloves on sale for 20% off their original price of $28. The next week the gloves have been reduced another 15% off the sale price. If there is a 6% sales tax, how much would the shopper now pay for the gloves?
A. $16.52  
B. $17.11  
C. $19.29  
D. $20.18

- Domain Four: Statistics, Data Analysis, and Probability

4.1 Collection, Organization, and Representation of Data

4A represent a collection of data through graphs, tables, or charts.

Use the diagram below to answer the question that follows:

The diagram above shows the number of music CDs owned by a sample of students in a middle school. What is the mean number of CDs owned by the students?

A. 2.50  
B. 2.71  
C. 2.83  
D. 3.40
6. Use the diagram below to answer the question that follows.

What percent of the entire rectangle shown above is shaded?
A. 5%
B. 7%
C. 20%
D. 25%

An elementary school is divided into three buildings: Building A, Building B, and Building C. In the school, 40% of the students are in Building A and 32% of the students are in Building B. When these data are presented in a circle graph, what is the measure of the central angle representing the number of students in Building C?
A. 28°
B. 72°
C. 100.8°
D. 129.6°

10. A high school gym class consists of 5 seniors, 7 juniors, 8 sophomores, and 6 freshmen. Two students will be chosen at random to assist the teacher in setting up equipment. What is the probability that a freshman will be chosen first and a senior chosen second?
A. \( \frac{2}{26} \)
B. \( \frac{11}{26} \)
C. \( \frac{30}{650} \)
D. \( \frac{30}{676} \)

The answer is C.
21. Students in Mr. Gonzales’s class have sorted and counted a collection of blocks by shape.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle</td>
<td>15</td>
</tr>
<tr>
<td>Rectangle</td>
<td>15</td>
</tr>
<tr>
<td>Triangle</td>
<td>10</td>
</tr>
<tr>
<td>Square</td>
<td>20</td>
</tr>
<tr>
<td>Total</td>
<td>60</td>
</tr>
</tbody>
</table>

The students would like to build a spinner to simulate the probability of randomly selecting a block of a given shape. Which of the following spinners could they use? ANS D
22. Miguel is playing a game called "Guess My Rule." Each time a classmate calls out a positive number (input number), Miguel adds 5 to the number and tells the class the result (output number). The game continues until a student is able to correctly predict Miguel's response and then state Miguel's rule. Which of the following graphs correctly shows some possible values of the input and output numbers for Miguel's rule? ANS D
31. Scientists have stocked Wilson's pond with a species of fish. The scientists note that the population has steadily decreased over a period of time until the population is approximately half the number of fish originally stocked. If the number of fish is plotted on the y-axis and the amount of time on the x-axis, which of the following graphs represents the population of fish in the pond with respect to time?

A

B

C

D

ANS A

4B understand the mean, median, mode, and range of a collection of data.

9. An individual makes seven measurements, rejects the lowest and highest, and uses the remaining five to do some statistical analyses. Which of the following statistics will remain unaffected by rejecting the lowest and highest measurements?
A. mean
B. median
C. range
D. standard deviation
20. Friendly Farms, Inc., has several farms. Over the years, it has been determined that the probability of sufficient rain during the maximum growth period is 25%. When there is not enough rain, Friendly Farms needs to supplement with irrigation. The profit when there is sufficient rain during the maximum growth period is $4000 per acre. If it does not rain during this period, the profit is only $1600 per acre. Using this information, what profit per acre, on average, can Friendly Farms expect to make?

A. $1000  
B. $2200  
C. $2800  
D. $3400  

In a unit on measures of central tendency, a middle school math teacher places 20 pennies on top of a straight edge. She then balances the straight edge on a triangular block. Which of the following measures of central tendency is most analogous to the point at which the straight edge balances?

A. variance  
B. mean  
C. mode  
D. median
Leah is going on a four-day vacation to a city that has a 30% chance of rain each day. To determine the probability that it will rain on two or more of the four days of her vacation, she creates the following simulation. Write the numbers 1 through 10 on slips of paper and put the slips in a box. Let the numbers 1, 2, and 3 represent rainy days. Without looking, draw a slip from the box, record the number, and throw the slip away. Repeat this procedure four times. If two or more of the four slips had the number 1, 2, or 3, count the trial as a positive result. Repeat this trial many times, and calculate the fraction of the trials that were positive.

For this to be an accurate simulation, it should be changed in which of the following ways?

A. After drawing a slip, replace the slip in the box.
B. Count the trial as a positive result if any of the four slips had the number 1, 2, or 3.
C. Do only a single trial, rather than many trials.
D. Count the trial as a positive result only if exactly two of the four slips had the number 1, 2, or 3.

Choose a Number
Add Five
Double the Result
Subtract Four
Divide by Two
Subtract the Original Number
The Result is Three

The information in the box above shows a student's explanation for why a given number trick works. In providing this explanation, the student has demonstrated an ability to:

A. prove a statement using deductive reasoning.
B. develop a conjecture using inductive reasoning.
C. solve a problem using proportional reasoning.
D. validate a statement using indirect reasoning.
Which of the following distributions corresponds to the data set with the largest standard deviation?

A. I  
B. II  
C. III  
D. IV

An economist is studying the relationship between the amount workers in a factory are paid per hour and the number of hours they choose to work per week. The economist's hypothesis is that as workers' hourly pay increases, they will increase the number of hours of work at a steady rate. For example, if a worker who works 30 hours per week has her pay raised from $9 to $10 per hour, she will work 32 hours per week; if her pay is then raised to $11 per hour, she will work 34 hours per week. In mathematical language, the economist's hypothesis is that the number of hours worked per week is:

A. a linear function of the worker's hourly wage.  
B. a constant multiple of the worker's hourly wage.  
C. a quadratic function of the worker's hourly wage.  
D. inversely proportional to the worker's hourly wage.
### 4.2 Inferences, Predictions, and Arguments Based on Data.

4D interpret a graph, table, or chart representing a data set.

<table>
<thead>
<tr>
<th>13. Use the pie charts below to answer the question that follows.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Pie Charts" /></td>
</tr>
<tr>
<td>The first pie chart represents a company's total expenditures, and the second pie chart shows a breakdown of the company's advertising expenditures.</td>
</tr>
<tr>
<td>What percent of the company's total expenditures is spent on radio advertising?</td>
</tr>
</tbody>
</table>
| **A.** 6.3%  
**B.** 11.7%  
**C.** 18.0%  
**D.** 35.0% |

<table>
<thead>
<tr>
<th>The plots above display state assessment results. Based on these plots, which of the following is an accurate statement to report to parents?</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Box Plot" /></td>
</tr>
<tr>
<td>Unfortunately, our scores this year were not as high as last year. This year a higher percentage of students than last year had raw scores below 72.</td>
</tr>
<tr>
<td><strong>D.</strong> Fortunately, our scores this year improved. Seventy-five percent of our students scored at or above last year's median.</td>
</tr>
</tbody>
</table>


24. Use the box and whisker diagram below to answer the question that follows.

<table>
<thead>
<tr>
<th>Test Scores</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
</tr>
</tbody>
</table>

The diagram above shows the distribution of scores of 135 psychology students on an 80-point midterm exam. Which of the following statements about the score distribution is true?

A. Fifty percent of all the scores fall between 33 and 55.
B. The average score is 44.
C. Fewer people achieved scores above 55 than below 33.
D. Fewer people achieved scores between 33 and 37 than between 37 and 55.

**Correct Response: A**. In a box and whisker diagram, the middle half of the data in a distribution is presented in a rectangle, or "box," whose vertical sides represent the first and third quartiles of the data. The vertical line segment inside the box represents the median of the data.

The "whiskers" are the horizontal lines outside of the box, and they include the range of the data. In the diagram in this problem, the scores are distributed from 0 to 75. Twenty-five percent of the scores are between 0 and 33, 25% are between 33 and 37, 25% are between 37 and 55, and 25% are above 55; hence choice A is the correct response.

There is not enough information in the plot to determine the average score (choice B). The same number of people achieved scores above 55 as below 33 (choice C), and the same number of people achieved scores between 33 and 37 as between 37 and 55 (choice D).
The numbers in the table represent the populations of two small towns, A and B, over a forty-year period. Which of the following statements is best supported by the data?

A. Town A and Town B are both growing more slowly in the 1990s than in the 1960s.
B. Town A initially grew more slowly than Town B but recently has been growing more rapidly.
C. Town A and Town B will eventually have equal populations.
D. Town A has consistently grown at a slower rate than Town B.

Correct Response: D. Although the rate of population change in each town has varied between 1960 and 2000, the percentage increase in population has been greater each ten years in Town B than the percentage increase in population in Town A. Thus, Town A has consistently grown at a slower rate than Town B.

<table>
<thead>
<tr>
<th>Year</th>
<th>Town A</th>
<th>Town B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>700</td>
<td>2900</td>
</tr>
<tr>
<td>1970</td>
<td>800</td>
<td>3700</td>
</tr>
<tr>
<td>1980</td>
<td>950</td>
<td>4600</td>
</tr>
<tr>
<td>1990</td>
<td>1100</td>
<td>5400</td>
</tr>
<tr>
<td>2000</td>
<td>1400</td>
<td>7100</td>
</tr>
</tbody>
</table>
22. The amount of money, after taxes, spent by a family on various expenses during one month is given in the table below.

<table>
<thead>
<tr>
<th>Type of expense</th>
<th>Amount spent in one month</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rent</td>
<td>$ 750</td>
</tr>
<tr>
<td>Food</td>
<td>$ 575</td>
</tr>
<tr>
<td>Utilities</td>
<td>$ 120</td>
</tr>
<tr>
<td>Car loan, gas, and repairs</td>
<td>$ 450</td>
</tr>
<tr>
<td>Medical expenses</td>
<td>$ 65</td>
</tr>
<tr>
<td>Entertainment</td>
<td>$ 120</td>
</tr>
<tr>
<td>Credit card payment</td>
<td>$ 350</td>
</tr>
<tr>
<td>Miscellaneous expenses</td>
<td>$ 95</td>
</tr>
<tr>
<td>Total expenses</td>
<td>$ 2525</td>
</tr>
</tbody>
</table>

If the family constructs a pie chart using these figures, what is the approximate measure of the central angle of the sector used to represent the percentage of total expenses spent on food?

A. 23°
B. 63°
C. 77°
D. 82°
1. In which of the states listed in the graph above did the cost of congressional campaigns increase the most between 1984 and 1994?
A. Arizona  
B. New York  
C. Nevada  
D. Pennsylvania

First examine the stimulus, a bar graph containing information on the cost of congressional campaigns in 1984 and 1994. Note how the information is presented. The vertical axis on the left provides information on campaign costs in dollars. The horizontal axis along the bottom identifies four different states. For each state there are two bars. The first bar indicates the average amount of money individual candidates spent on congressional campaigns during 1984. The second bar indicates the average amount of money individual candidates spent on congressional campaigns in 1994. Now you are prepared to address the sample test questions associated with this stimulus. Both of the sample test questions will probably require you to look back at the graph to determine the correct response. Sample test question number 1 asks you to determine in which of the four states listed on the graph did campaign costs rise the most between 1984 and 1994. By using the dollar amounts indicated on the vertical axis of the graph, you can see that the average cost of congressional campaigns went from about $150,000 to about $200,000 in Arizona, from about $450,000 to about $750,000 in New York, from about $100,000 to about $140,000 in Nevada, and from about $250,000 to about $400,000 in Pennsylvania. By comparing the percentage increase in each state, you will find that average campaign costs during this period rose by 33 percent in Arizona, by 67 percent in New York, by 40 percent in Nevada, and by 59 percent in Pennsylvania. Therefore, the correct response is choice B, New York.
Based on the information contained in the graph, which of the following is most likely to be true of congressional campaigns in the year 2004?

A. The cost of congressional campaigns in Arizona and Nevada will be approximately $300,000.

B. The gap between the cost of congressional campaigns in urban northeastern states and the cost of congressional campaigns in rural western states will be greater than it was in 1994.

C. The cost of congressional campaigns in New York and Pennsylvania will be about twice what it was in 1994.

D. The gap between the cost of congressional campaigns in urban northeastern states and the cost of congressional campaigns in rural western states will be less than it was in 1994.

Sample test question number 2 asks you to use the information presented in the graph to predict the cost of congressional campaigns in the future. Specifically, the question asks you to consider which of four predictions about the election of 2004 best accords with the changes in the cost of congressional campaigns reflected in the graph. According to the graph, the costs of the 1994 congressional campaigns in Arizona and Nevada were about $200,000 and $140,000, respectively. If the costs of financing campaigns in those states increase at the same rates as they did between 1984 and 1994, campaign costs in 2004 would still be well below the $300,000 predicted by choice A. The graph clearly does not support the idea that over the next ten years campaign costs in New York and Pennsylvania will double, as predicted in choice C. The fact that the rate of increase in campaign costs between 1984 and 1994 was greater in New York and Pennsylvania than it was in Arizona and Nevada, however, suggests that the gap between those states will increase, not decrease, as predicted in choice D. Therefore, the correct response is choice B.

32. Use the distribution curves below to answer the question that follows.

The distribution curves above show data on the gas mileage for two different brands of car. Which of the following correctly analyzes the information presented in these distributions?

A. The mean gas mileage of Brand A is greater than the mean gas mileage of Brand B.

B. Data was collected for more cars of Brand A than of Brand B.

C. Brand A cars have smaller variability in gas mileage than Brand B cars.

D. Brand A cars get poorer gas mileage than Brand B cars.
24. Use the table below to answer the question that follows.

<table>
<thead>
<tr>
<th>Cuts of Beef</th>
<th>High Fat Content</th>
<th>Low Fat Content</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flank Steaks</td>
<td>74</td>
<td>386</td>
<td>460</td>
</tr>
<tr>
<td>Rump Roasts</td>
<td>258</td>
<td>142</td>
<td>400</td>
</tr>
<tr>
<td>Total</td>
<td>332</td>
<td>528</td>
<td>860</td>
</tr>
</tbody>
</table>

A USDA inspector is grading cuts of beef at a meat packing plant. If a piece of beef is selected at random, what is the probability that it will be a flank steak with high fat content?

A. 0.0860  
B. 0.1609  
C. 0.2229  
D. 0.3860

25. A sixth grade student attains a grade equivalent score of 8.6 on a sixth grade standardized reading achievement test administered in February. A correct interpretation of this score is that the sixth grader:

A. achieved a score that eighth graders would be expected to achieve on a grade-appropriate standardized reading test administered in April.
B. can do eighth grade work as well as the average eighth grade student in the sixth month.
C. performed as well as students at grade level 8.6 would perform if they took the same test that the sixth grader took.
D. reads as well as 80 percent of eighth graders.

Correct Response: C. Grade equivalent scores are used to describe a student's academic achievement in terms of grade levels and to measure changes in achievement levels from one academic year to the next. Grade equivalent scores are expressed in whole grade numbers and to the nearest tenth, with each month corresponding to one-tenth of the academic year. Therefore, a sixth grade student who tested in February (the sixth month of the academic year), and has received a grade equivalent score of 8.6, is performing approximately equal to that of an eighth grade level student who tested at the same time of year (choice C). The score expected of eighth grade students tested in April would be approximately 8.8 (choice A). If a student has received a particular grade equivalent score, it does not necessarily mean that he or she can do eighth grade work (choice B). Grade equivalent scores do not represent achievement levels attained by a percentage of a population group (choice D).
26. A company claims that less than 2% of its electronic components are faulty. Tests on a random sample of 50 components find 4 faulty components. How would statistical principles be used to evaluate the company's claim?

A. by repeating the tests on a random sample of 25 components to see if 2 faulty components are found

B. by examining the components to determine the reason for their failure

C. by assuming the sample is not random and rejecting the results of the test

D. by determining the probability of finding 4 faulty components in a random sample of 50

4. Which of the following situations best represents a random sampling?

A. Ask every tenth person coming out of a health spa how many times a week they exercise to determine how often people in the town exercise.

B. Survey students in advanced biology classes to determine the average amount of time students in a certain school study each week.

C. Find the heights of all boys in a senior gym class to determine the average height of all boys in the school.

D. Count the number of chocolate chips in every fifth cookie to determine the average number of chocolate chips per cookie in a bag of cookies.
4.3 Basic Notions of Chance and Probability

4F define the concept of probability in terms of a sample space of equally likely outcomes.

<table>
<thead>
<tr>
<th>Use the information below about the members of a set to answer the question that follows.</th>
</tr>
</thead>
<tbody>
<tr>
<td>• No odd numbers are in the set.</td>
</tr>
<tr>
<td>• All members of the set are greater than 34.</td>
</tr>
<tr>
<td>• If a number is in the set, it is a multiple of 5.</td>
</tr>
</tbody>
</table>

Which of the following is a valid conclusion from the above premises?

A. The set is empty.
B. All members of the set are greater than 39.
C. The set does not include 90.
D. All multiples of 5 that are greater than 34 are in the set

4. A student rolls two dice. One die has six faces, each face marked with 1, 2, 3, 4, 5, or 6 spots, and the other die has eight faces, each marked with 1, 2, 3, 4, 5, 6, 7, or 8 spots.

How many outcomes are in the sample space for this experiment?

A. 14
B. 28
C. 48
D. 64

4G use their understanding of complementary, mutually exclusive, dependent, and independent events to calculate probabilities of simple events

<table>
<thead>
<tr>
<th>Which of the following statements is reasonable?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. The volume of a car is about two hundred cubic meters.</td>
</tr>
<tr>
<td>B. A bicycle weighs about five hundred grams.</td>
</tr>
<tr>
<td>C. The perimeter of a classroom is about a million centimeters.</td>
</tr>
<tr>
<td>D. A man has lived for over a billion seconds.</td>
</tr>
</tbody>
</table>
10. Use the table below to answer the question that follows.

<table>
<thead>
<tr>
<th>Year</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50,000</td>
</tr>
<tr>
<td>1</td>
<td>51,000</td>
</tr>
<tr>
<td>2</td>
<td>52,020</td>
</tr>
<tr>
<td>n</td>
<td>?</td>
</tr>
</tbody>
</table>

The table gives the population of a town over a three-year period. If the population continues to increase at the same rate per year, which of the following equations could be used to predict the population, \( P(n) \), in the \( n \)th year?

A. \( P(n) = 50,000 + 500n \)
B. \( P(n) = 50,000(0.02)n \)
C. \( P(n) = 50,000 + 1,000n \)
D. \( P(n) = 50,000(1.02)n \)
4H can express probabilities in a variety of ways, including ratios, proportions, decimals, and percents.

A basketball player shoots foul shots with a .70 probability of scoring. What is the probability that the basketball player will make only one of the next two foul shots attempted if it is assumed that the outcomes of the foul shots are independent?

A. 0.30  
B. **0.42**  
C. 0.49  
D. 0.60

Correct Response: **B**. The probability that two independent events will occur is the product of their individual probabilities. Mathematically, this is expressed \( P(A \cap B) = P(A) \cdot P(B) \). In the situation described in this problem, four mutually exclusive outcomes are possible, as summarized in the following chart:

<table>
<thead>
<tr>
<th>Outcome</th>
<th>First Foul Shot ( P(A) )</th>
<th>Second Foul Shot ( P(B) )</th>
<th>Product of Probabilities ( P(A \cap B) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>successful (0.70)</td>
<td>successful (0.70)</td>
<td>0.49</td>
</tr>
<tr>
<td>II</td>
<td>successful (0.70)</td>
<td>not successful (0.30)</td>
<td>0.21</td>
</tr>
<tr>
<td>III</td>
<td>not successful (0.30)</td>
<td>successful (0.70)</td>
<td>0.21</td>
</tr>
<tr>
<td>IV</td>
<td>not successful (0.30)</td>
<td>not successful (0.30)</td>
<td>0.09</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>

Since the problem does not specify which shot is successful and which is not, outcomes II and III both satisfy the condition that the player makes "only one of the next two foul shots attempted."

Mathematically, this is expressed \( P(A \cap B) = P(A) + P(B) \). Therefore, the probability of success on one of the next two shots is the sum of the probabilities of outcomes II and III, or 0.42. Choice A represents the probability of any one shot being unsuccessful. Choice D is derived as a result of doubling the answer in choice A. Choice C represents the probability of two successful shots.

Three partners own a restaurant. If the first partner owns a \( \frac{1}{3} \) share and the second partner owns a \( \frac{1}{4} \) share, how much of the restaurant does the third partner own?

A. \( \frac{5}{12} \)  
B. \( \frac{1}{2} \)  
C. \( \frac{7}{12} \)  
D. \( \frac{3}{4} \)

The correct response is A. First, you must add the number of shares the two known partners own. Second, you must subtract them from the whole (1). To perform the first step of adding the fractions, \( \frac{1}{3} \) and \( \frac{1}{4} \), converted them to a common denominator.

Since \( \frac{1}{3} = \frac{4}{12} \) and \( \frac{1}{4} = \frac{3}{12} \), we will add \( \frac{4}{12} \) and \( \frac{3}{12} \) to get \( \frac{7}{12} \). This completes step one in the process. Our second step is to subtract \( \frac{7}{12} \) from 1:

\[
\frac{1 - 7}{12} = \frac{12 - 7}{12} = \frac{5}{12}
\]

The correct answer is A.
20. A student received a grade of 64% on a test weighted as 25% of the final grade, a grade of 80% on a test weighted as 35% of the final grade, and a grade of 70% on a test weighted as 20% of the final grade. The student has not yet taken the remaining test weighted as 20% of the final grade. What is the student's current percentage grade?

A. 71.5%
B. 72.5%
C. 73.5%
D. 74.5%

Correct Response: B. The student has earned $25(0.64) + 35(0.80) + 20(0.70) = 58$ of the 80 percentage points at this time. Since $\frac{58}{80} = 0.725$, the student's current percentage grade average is 72.5.

7. A high school gym class consists of 5 seniors, 7 juniors, 8 sophomores, and 6 freshmen. Two students are chosen at random to assist the teacher with setting up equipment. What is the probability that a freshman will be chosen first and a senior chosen second?

A. $\frac{2}{26}$
B. $\frac{11}{26}$
C. $\frac{30}{650}$
D. $\frac{30}{676}$

Correct Response: C. $\frac{30}{650}$

4. A solution of salt and water is 80% water by weight. After a period of time, some of the water evaporates and the solution is 40% water by weight. What is the ratio of the final weight of water to the initial weight of water in the mixture?

A. $\frac{1}{6}$
B. $\frac{2}{10}$
C. $\frac{1}{3}$
D. $\frac{1}{2}$

Correct Response: D. $\frac{1}{2}$
A student is playing a game with a set of cards on which are printed the 21 consonants and 5 vowels that compose the English alphabet. The student turns the cards face down, shuffles them, and draws a card from the top of the stack. Given that the student has drawn a vowel, what is the probability that the letter is $A$ or $E$?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A.</strong></td>
<td>$\frac{1}{26}$</td>
</tr>
<tr>
<td><strong>B.</strong></td>
<td>$\frac{1}{13}$</td>
</tr>
<tr>
<td><strong>C.</strong></td>
<td>$\frac{1}{5}$</td>
</tr>
<tr>
<td><strong>D.</strong></td>
<td>$\frac{2}{5}$</td>
</tr>
</tbody>
</table>

8. Four families are picnicking with their children. The Chavez family has 2 children, the Wilsons have 3, the Pratts have 4, and the Shumway family has 6 children. If the children line up to get their lunches in a random order, what is the probability that the first child to get a lunch would be a Pratt?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A.</strong></td>
<td>$\frac{1}{15}$</td>
</tr>
<tr>
<td><strong>B.</strong></td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td><strong>C.</strong></td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td><strong>D.</strong></td>
<td>$\frac{4}{11}$</td>
</tr>
</tbody>
</table>

Answer C

Answer D.
25. When a student is questioned about his school, he replies that there are at least as many freshmen as there are juniors and at least as many juniors as there are sophomores. If the student is correct, which of the following statements must be true?
A. There are just as many sophomores as there are freshmen.
B. There are at least as many sophomores as there are freshmen.
C. There are at least as many freshmen as there are sophomores.
D. There are more freshmen than there are sophomores.

Two tickets are drawn with replacement from a box containing four tickets numbered 1 through 4. What is the probability that the product of the numbers on the two tickets is 9 or greater?
A. \( \frac{1}{4} \)
B. \( \frac{1}{3} \)
C. \( \frac{1}{2} \)
D. \( \frac{3}{4} \)
A public opinion pollster is estimating the percentage of likely voters in four towns by interviewing a random sample of people in each town. The population of each town and the size of the sample of people interviewed for each poll are given in the table below.

<table>
<thead>
<tr>
<th>Poll</th>
<th>Population of Town</th>
<th>Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>92,000</td>
<td>500</td>
</tr>
<tr>
<td>2</td>
<td>60,000</td>
<td>750</td>
</tr>
<tr>
<td>3</td>
<td>35,000</td>
<td>600</td>
</tr>
<tr>
<td>4</td>
<td>25,000</td>
<td>400</td>
</tr>
</tbody>
</table>

Which poll has the highest probability of correctly estimating the percentage of likely voters?
A. Poll 1  
B. Poll 2  
C. Poll 3  
D. Poll 4

23. Two teams meet in a playoff series at the end of the regular season. Team A won 55 of 81 games played in its home stadium during the regular season, while Team B won 48 of 81 games played in its home stadium. The first two games of the series will be played in Team A’s home stadium, the next two games in Team B’s home stadium. In the absence of any other information, which expression is equal to the probability that Team A will win the first four games in a row? ANS C

A. \(\frac{55}{81} \times \frac{55}{81} \times \frac{55}{81} \times \frac{55}{81}\)  
B. \(\frac{55}{81} \times \frac{55}{81} \times \frac{48}{81} \times \frac{48}{81}\)  
C. \(\frac{55}{81} \times \frac{55}{81} \times \frac{33}{81} \times \frac{33}{81}\)  
D. \(\frac{55}{81} \times \frac{55}{81} \times \frac{26}{81} \times \frac{26}{81}\)
Domain Two: Algebra and Functions

2.1 Patterns and Functional Relationships.

2A represent patterns, including relations and functions, through tables, graphs, verbal rules, or symbolic rules.

8. If $y$ is inversely proportional to $x$ and $y = \frac{1}{3}$ when $x = 360$, what is the value of $y$ when $x = 540$?

A. $\frac{2}{9}$

B. $\frac{1}{2}$

C. 2

D. $\frac{9}{2}$

The correct response is A. In an inverse proportion, one term of the function increases as the other term decreases. A general equation of an inverse proportion is $y = \frac{k}{x}$, where $k$ is the constant proportion and $y$ and $x$ are the terms that are inversely related. You can find the constant of proportionality for this problem by simply substituting $\frac{1}{3}$ for $y$ and 360 for $x$.

\[
y = \frac{k}{x}
\]

\[
\frac{1}{3} = \frac{k}{360}
\]

\[
3k = 360
\]

\[
k = 120
\]

The value of $y$ when $x$ is equal to 540 can now be found using 120 as the constant of proportionality.

\[
y = \frac{k}{x}
\]

\[
\frac{120}{540} = \frac{2}{9}
\]
Students measure the circumference and diameter of a number of circular objects and graph the circumference on the y-axis and the diameter on the x-axis. The students then draw a line that seems to best fit the data points. How can the students use their graphs to estimate the value of \( \pi \)?

A. find the length of the line  
B. **find the slope of the line**  
C. find the \( y \)-intercept of the line  
D. find the midpoint of the line

Which of the following situations is best modeled by a relation of the form \( y = kx \) where \( y \) and \( x \) are variables and \( k \) is a constant?

A. John needs to travel 5 miles. He could walk, ride his bike, or take a taxi. How does the length of travel time depend on the speed of travel?  
B. **A store is having a sale in which all items in the store are** 80% of the regular price. How does the discounted price of an item depend on the regular price of the item?  
C. Students are cutting out square pieces of cardboard. How does the area of each square depend on the length of the side of the square?  
D. A plumbing service charges $40 per hour and a fixed fee of $15 for house calls. How does the total cost of a house call depend on the number of hours needed to fix the plumbing problem?
The graph shown above represents the relationship between the length of a geyser's eruption, B, and the time until the next eruption, t. Which of the following linear equations best models the data?

A. $t + 12B = 33$
B. $2t - 3B = -66$
C. $12B - t = -33$
D. $3B + 2t = 66$
use proportional reasoning such as ratios, equivalent fractions, and similar triangles, to solve numerical, algebraic, and geometric problems.

2.2 Linear and Quadratic Equations and Inequalities

find equivalent expressions for equalities and inequalities, explain the meaning of symbolic expressions (e.g., relating an expression to a situation and vice versa).

Find the quotient.

\[
\frac{x^2 - x}{x^2 + 3x + 2} \div \frac{x^2 + 2x}{x^2 - 1}
\]

A. \( \frac{x^2}{x^2 + 2x + 1} \)

B. 1

C. \( \frac{x^2 - 1}{x^2 - 4} \)

D. \( \frac{x^2 - 2x + 1}{x^2 + 4x + 4} \)

D is the correct response.

To find the quotient, invert the second rational expression, factor, simplify, and multiply out as follows:

**Begin** with the question

\[
\frac{x^2 - x}{x^2 + 3x + 2} \div \frac{x^2 + 2x}{x^2 - 1} = \]

**Invert** the second equation

\[
\frac{x^2 - x}{x^2 + 3x + 2} \div \frac{x^2 - 1}{x^2 + 2x} = \]

**Factor** each expression (e.g., get rid of all of the squares)

\[
\frac{x^2 - x}{x^2 + 3x + 2} \div \frac{x^2 - 1}{x^2 + 2x} = (becomes) \]

\[
\frac{x(x - 1)}{(x + 2)(x + 1)} \div \frac{(x + 1)(x - 1)}{x(x + 2)} \]

**Simplify** each expression by cross-canceling where possible:

\[
\frac{x(x - 1)}{(x + 2)(x + 1)} \div \frac{(x + 1)(x - 1)}{x(x + 2)} = \]

**Multiply** the simplified expression out and arrive at the answer:

\[
\frac{(x - 1)(x - 1)}{(x + 2)(x + 2)} = \frac{x^2 - 2x + 1}{x^2 + 4x + 4} \]
Which of the following is equivalent to the expression below?

\[
\frac{x + 2}{x + 4} + \frac{x}{x + 3}
\]

A. \(\frac{2x + 2}{2x + 2} \frac{2x^2 + 9x + 6}{x^2 + 7x + 12}\)

B. \(\frac{2x^2 + 10}{x^2 + 12}\)

C. \(\frac{2x^2 + 5x + 10}{x^2 + 7x + 12}\)

D. \(\frac{2x^2 + 9x + 6}{x^2 + 7x + 12}\)

ANS D

The correct answer is D.

\[
\frac{2x^2 + 9x + 6}{x^2 + 7x + 12}
\]

7. An insurance agency earns $400,000 in revenue per year from existing accounts and estimates that it can earn another $250,000 per year in revenue for each salesperson it has on staff. The agency also estimates that an additional $1.10 can be earned for every dollar it spends on advertising. The agency’s fixed annual operating costs, not including advertising, are $800,000 for office space and equipment, plus $150,000 per salesperson for salary, commissions, benefits, expenses, and office space.

If \(s\) represents the number of salespeople, and \(a\) represents the amount spent on advertising, which of the following sentences models the combinations of salespeople and advertising that would yield a net profit?

A. \(650,000s + 1.10a > 800,000 + 150,000s + a\)

B. \(400,000 + 250,000s + 1.10a > 950,000s + a\)

C. \(400,000 + 250,000s + 1.10a > 800,000 + 150,000s + a\)

D. \(400,000 + 250,000s + 1.10a > 800,000 + 150,000s\)

Correct Response: C. The total revenue earned by the insurance agency includes $400,000 from existing accounts, $250,000s from revenues brought in by salespeople, and $1.10a from revenues brought in by advertising. The total cost includes $800,000 for fixed operating costs; $150,000s for salespeople’s salaries, commissions, benefits, expenses, and office space; and $a for advertising. Therefore, the total revenue in dollars is \(400,000 + 250,000s + 1.10a\), and the total cost is \(800,000 + 150,000s + a\). A net profit will be earned when total revenue is greater than total cost, or when \(400,000 + 250,000s + 1.10a > 800,000 + 150,000s + a\), which is choice C. Choice A results from assuming that both the total revenue from existing accounts and the revenue brought in from salespeople depend on the number of salespeople.

Choice B results from assuming that both the fixed operating costs and the costs associated with each salesperson depend on the number of salespeople. Choice D results from neglecting to take into account advertising costs.
Use the diagram below of a balance scale to answer the question that follows.

Each of the rectangles represents a 1-gram weight. The weight of the triangles is unknown, but they all have the same weight. If \( x \) stands for the weight of the triangles, which of the following correctly represents the relationship between the two sides of the balance scale?

A. \( 5x + 3 < 2x + 4 \)
B. \( 8x - 3 > 6x - 4 \)
C. \( 3x + 5 < 4x + 2 \)
D. \( 8x > 6x \)

Correct Response: A. The weight of a rectangle is 1 gram, so the weight of the rectangles can be represented by whole numbers (i.e., 3 on the left side of the balance scale and 4 on the right). Because the weight of a triangle is unknown, it is represented by the variable \( x \). Thus, the weight of the 5 triangles on the left side of the balance scale should be represented by \( 5x \) and the weight of the 2 triangles on the right side of the balance scale should be represented by \( 2x \). Finally, the balance scale is tipped to the right, indicating that the objects on the left weigh less than the objects on the right. This relationship should be represented by the sign \(<\).

\[
\begin{array}{|c|c|}
\hline
\chi^2 & 8x \\
\hline
2x & 16 \\
\hline
\end{array}
\]

The diagram above is a geometric representation of which of the following operations?
A. the sum of \((x + 4)^2\) and \(10x\)
B. the difference of \((x - 4)^2\) and \(2x\)
C. the product of \((x + 2)\) and \((x + 8)\)
D. the quotient of \(\chi^2 + 16\) and \(10x\)
Use the inequality below to answer the question that follows:

\[-6 \leq \frac{x}{3} - 3 \leq 5\]

Which of the following best represents the solution of the inequality above

A.  
B.  
C.  
D.  

ANS D

The correct answer is D.

Use the calendar below to answer the question that follows.

![Calendar Image]

A student notices that in any two-by-two box of four dates on a calendar, the two diagonals of the box sum to the same number. If $n$ represents the largest number in a two-by-two box of four dates, which of the following equations expresses this relationship?

A. $n + (n - 7) = (n - 1) + (n - 6)$  
B. $n + (n - 7) = (n - 1) + (n - 6)$  
C. $n + (n - 8) = (n - 1) + (n - 7)$  
D. $n + 2(n - 8) = (n - 1) + 2(n - 7)$
Students in a fourth-grade class are given the following problem.

*A farmer would like to enclose a 24-square-foot rectangular region with fencing. If the length and width of the rectangle are whole numbers, what is the least amount of fencing the farmer could use?*

This problem would be most appropriate to use in a unit addressing which of the following?

**A. Find factor pairs and products using arrays and area models.**

**B. Make generalizations from patterns and sets of examples.**

**C. Use multiplication to solve problems involving two-digit numbers.**

**D. Describe shapes and solids in terms of vertices, edges, and faces.**

2D find the solutions, and represent them on graphs.

**2. Use the diagram below to answer the question that follows.**

ANS B
30. Use the pie charts below to answer the question that follows.

The first pie chart represents a company's total expenditures, and the second pie chart shows a breakdown of the company's advertising expenditures. What percentage of the company's total expenditures is spent on radio advertising?

A. 6.3%
B. 11.7%
C. 18.0%
D. 35.0%

What is the domain of the function machine shown above?

A. all real numbers
B. all real numbers greater than -4
C. all real numbers except 2 and -2
D. all real numbers except 4 and -4
Recognize and create equivalent algebraic expressions (e.g., $2(a+3) = 2a + 6$)

Solve for $x$.

\[ \frac{1}{x-3} = \frac{6}{(x-3)(x+3)} + \frac{8}{7} \]

D is the correct response. A fast method for solving this rational equation is to multiply each term by the lowest common denominator to get rid of all of the fractions. The lowest common denominator is: $(7)(x – 3)(x + 3)$:

\[
\begin{align*}
\text{Pose the common denominator:} \\
(7)(x – 3)(x + 3) & \left[ \frac{1}{x-3} = \frac{6}{(x-3)(x+3)} + \frac{8}{7} \right] \\
\text{Cancel (x – 3) from the left and from the bracketed equation:} \\
(7)(x – 3)(x + 3) & \left[ \frac{1}{x-3} = \frac{6}{(x-3)(x+3)} + \frac{8}{7} \right] \\
\end{align*}
\]

By multiplying $(7)(x – 3)(x + 3)$ and cross-canceling like terms, we are left with the following equation: We continue multiplying where possible:

\[ (7)((x + 3) = (7) (6) + (8) (x-3) (x+3) \]

\[ 7x + 21 = 42 + x^2 – 72 \]

Continue reducing where possible:

\[ 7x + 21 = 8x^2 – 30 \]

Collect like terms on one side of the equation and set it equal to zero.

\[ 8x^2 – 7x – 51 = 0 \]

Factor the equation:

\[ (8x+17)(x-3) = 0 \]

Set each equation equal to zero:

\[ 8x + 17 = 0 \quad \text{or} \quad x – 3 = 0 \]

Solve each equation:

\[ 8x = -17 \quad \text{or} \quad x = 3 \]

\[ x = \frac{-17}{8} \]

At last, check each possible solution in the original equation. The solution $x = 3$ gives zero as a denominator in this equation, and that means option C is out. Only $\frac{-17}{8}$ is an appropriate solution, making choice D the only correct response. Choice B is wrong because 8 is ignored in solving $8x +17 = 0$. Choice A represents the same error, plus a failure to check for denominators equal to zero.
2F represent geometric problems algebraically (e.g., the area of a triangle).

Rectangles $ABCD$ and $BCEF$ are similar, $AF = AD$, and $EF$ is perpendicular to $AB$. If $AB = 1$ and $FB = x$, which of the following equations must be true?

A. $x^2 - x + 1 = 0$
B. $x^2 - 2x - 2 = 0$
C. $x^2 - 2x + 2 = 0$
D. $x^2 - 3x + 1 = 0$

Correct Response: B. The area of rectangle $EFGH$ is equal to $a(b + c)$. The area of rectangle $EIJH$ is equal to $ab$, and the area of rectangle $IFGJ$ is equal to $ac$. Setting the areas above equal to each other results in the equation $a(b + c) = ab + ac$, which is the distributive property of the real numbers.

Choice A is incorrect because the associative property of multiplication describes a relationship about the grouping of the factors in a product.

Choice C is incorrect because the inverse property of addition states that for any number $x$, $x + -x = 0 = -x + x$, a relationship that is not used in this situation.

Choice D is incorrect because the associative property of addition describes a relationship about the grouping of three or more numbers in a sum.
4. Use the diagram below to answer the question that follows.

A dog kennel owner plans to build five adjacent rectangular running pens out of 150 meters of fencing. If each pen measures $x$ meters by $y$ meters, with a total area of 468 square meters for the five pens, which of the following quadratic equations can be used to determine the value of $x$?

A. $6x^2 - 150x + 468 = 0$
B. $3x^2 - 75x + 468 = 0$
C. $x^2 - 75x + 468 = 0$
D. $x^2 - 150x + 468 = 0$

Correct Response: B. To build the dog pens as described, the dog kennel owner would need 10 pieces of fencing $y$ meters long (5 pieces for each end) and 6 pieces $x$ meters long. Therefore, $10y + 6x = 150$. Since the total area is the product of the length and the width, $468 = x(5y)$. The first equation is equivalent to $5y = 75 - 3x$. Substituting this equation into the expression for the area results in $468 = x(75 - 3x)$, which leads to the quadratic equation $3x^2 - 75x + 468 = 0$.

1. Use the information below to answer the question that follows.

The area of rectangle $EFGH$ can be expressed as the sum of the areas of rectangles $EIJH$ and $IFGJ$. This relationship is a geometric representation of which of the following properties of the real numbers?

A. associative property of multiplication
B. distributive property
C. inverse property of addition
D. associative property of addition

Correct Response: B. A set of algebra tiles usually consists of large squares that measure $x$ by $x$, rectangles that measure $x$ by 1, and small unit squares that measure 1 by 1. The area of these squares can be used to represent quadratic expressions since the area of a large square equals $x^2$, the area of a rectangle equals $x$, and the area of a unit square equals 1. In the diagram given, there are one large square, four rectangles, and three unit squares. Hence, the tiles represent the quadratic expression $x^2 + 4x + 3$. This is also equivalent to $(x + 3)(x + 1)$, the product of the side lengths of the given figure.
A dog kennel owner plans to build five adjacent rectangular running pens out of 150 meters of fencing. If each pen measures $x$ meters by $y$ meters, with a total area of 468 square meters for the five pens, which of the following quadratic equations can be used to determine the value of $x$?

A. $x^2 - 150x + 468 = 0$
B. $x^2 - 75 + 468 = 0$
C. $3x^2 - 75x + 468 = 0$
D. $6x^2 - 150x + 468 = 0$

Correct Response: C

Your task in this question is to determine which of the following quadratic equations can be used to determine the value of $x$? This question is not easy to solve, and I would be sure to skip it in favor of much easier questions to answer. But just in case you find yourself in a position to answer a question like this, let's look at how to do it.

You are given the following information:

- 468 is the total area of the field
- you have 150 yards of available fencing
- You don’t know what the length of the field is ($x$)
- You don’t know what the width ($y$)

There are actually two equations here. The first is the total area of the field. The second is the area of the fencing of each pen. Let’s look at each one.

**Total Area of the Field**

Set up the first equation to equal the number of known yards (468) for the 5 pens. Since the total area of any rectangle is the product of the length and the width, the equation will look like this

$$468 = 5yx.$$ 

meaning that the total area of 468 is equal to 5 pens times the length of $x$ times $y$. That is the easy part.
The Total Length of Fencing
Look at the diagram and remember that you’re building pens. You’re going to need 6 pieces of x and 10 pieces of y (5 pieces of y at each end) and 6 pieces. Remember that there are 150 meters of fencing available. Therefore,

\[ 150 = 10y + 6x \]

or 10 pieces of fencing times something plus 6 pieces of fencing times something equals 150 meters of available fencing.

Setting Up the Quadratic Equation
Which of the following quadratic equations can be used to determine the value of x? is the question we’re trying to answer. You now have two equations to work with:

\[
\begin{align*}
468 &= 5yx. \\
150 &= 10y + 6x
\end{align*}
\]

Look at the first equation. \[468 = 5xy\] is the same as \[468 = (5y)(x)\]. Look at the second equation: \[150 = 10y + 6x\]. If you can find a way to get \(5y\) by itself from the \[150 = 10y + 6x\] equation, then you can substitute it into \[468 = (5y) (x)\]. Here is how to do that.

Subtract \(-6x\) from each side:

\[
\begin{align*}
150 &= 10y + 6x \\
-6x &= -6x
\end{align*}
\]

This results in the following expression

\[10y = 150 - 6x\]

To get \(5y\), divide each side by 2:

\[
\left( \frac{1}{2} \right) 10y = \left( \frac{1}{2} \right) (150 - 6x)
\]

The results in the following expression:

\[5y = 75 - 3x\]

Because you now have \(5y = 75 - 3x\), you can substitute it into \(468 = 5yx\), where \(5y\) will equal \(17 - 3x\).

\[468 = 5yx.\]

\[468 = (5y) (x).\]

\[468 = (75 - 3x) (x)\]
Multiply \( (x) \) times \((75 - 3x)\) in the equation \(468 = (75 - 3x) \cdot (x)\):

\[
468 = 75x - 3x^2
\]

Since it is a quadratic equation, you must set it at zero. Do so by subtracting \(75x - 3x^2\) from each side and arranging the terms into the proper form.

\[
\begin{align*}
+468 &= 75x - 3x^2 \\
-75x + 3x^2 &= -75x + 3x^2 \\
-75x + 3x^2 + 468 &= 0
\end{align*}
\]

Write the equation in standard quadratic form, with the terms arranged properly, and you have:

\[
3x^2 - 75x + 468 = 0 \text{ or Choice C.}
\]

Choice A results from calculating the total amount of fencing as \(5y + x\).

Choice B results from calculating the total amount of fencing as \(10y + 2x\).

Choice D results from calculating the total amount of fencing as \(5y + 6x\).

The correct response is B.

This diagram is made up of squares and rectangles, one large square, three small squares, and four rectangles.
Remember that the formula for calculating the area of either a square or a rectangle is side times side, so we have a number of possibilities:

- 1 large square of x times x ($x^2$)
- 3 smaller squares of 1 times 1
- 4 rectangles of 1 times x. (1x)

If you add up the number of large squares, small squares, and rectangles, you arrive at the following:

- $x^2$ = One Large Square
- 3 is the sum of the three small squares
- 4x is the sum of the rectangles (1x + 1x + 1x + 1x)

**The Quadratic Equation**
Because the area of a large square equals $x^2$, we can apply the quadratic expression.

$$x^2 + x + n = 0$$

Simply plug in the known information into the quadratic expression

$$x^2 + 4x + 3$$

This also matches answer C.

**A Short Cut**
Let’s look at the question one more time.
Note that this expression is also equivalent to \((x + 3)(x + 1)\), and is also the short cut to solving this question. Next time you encounter a question like this, simply remember that area is simply side times side. Looking at the diagram below, you can see that one side is equal to \(x + 1\) and the other side is equal to \(x + 1 + 1 + 1\). Plug these numbers into our area = side times side formula and it looks like this:

The result of multiplying \((x + 3)(x + 1)\) is \(x^2 + 4x + 3\).

Use the diagram below to answer the question that follows:

Which of the following expression is represented by the areas of the rectangles in the diagram above?

A. \(xy^3 + xy^2\)
B. \(x^2 + 5xy + 6y^2\)
C. \(2x + y^3 + y^2\)
D. \(2x + 5xy + 6y\)

ANS B

\((x + 2y)(x + 3y)\)
14. If $6b + 20 = a$, and $4b + 30 = a$, then $b =$
A. $-5$
B. $-1$
C. 1
D. 5

The answer is D. To solve this bit of misery, separate the equations and plug the numbers in until you find the equivalency:

<table>
<thead>
<tr>
<th>$6b + 20$</th>
<th>$4b + 30$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>-10</td>
</tr>
<tr>
<td>-1</td>
<td>14</td>
</tr>
<tr>
<td>1</td>
<td>26</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
</tr>
</tbody>
</table>
26. Use the graph below to answer the question that follows.

![Graph showing the height of a candle over time](image)

The graph shows data collected by measuring the height, \( h \), in centimeters, of a burning candle at different times, \( t \), in minutes.

Which of the following equations best represents the line drawn through the data points?

<table>
<thead>
<tr>
<th>Option</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( h = -2t + 10 )</td>
</tr>
<tr>
<td>B</td>
<td>( h = \frac{1}{2}t + 10 )</td>
</tr>
<tr>
<td>C</td>
<td>( h = 2t + 20 )</td>
</tr>
<tr>
<td>D</td>
<td>( h = 10t + 20 )</td>
</tr>
</tbody>
</table>

B. \( h = \frac{1}{2}t + 10 \)

Notice that the height of the candle decreases over time. Thus, some subtraction must take place. This subtraction occurs by the negative numbers in answers A and B. Since no subtraction occurs in C and D, they are automatically out (the candle would grow over time, which is not what is happening here – it is burning).

The choice is between items A and B. Making the choice is simple. Plug in simple numbers, like 5 and 10 from the time line and see what happens. In answer A, for example, the candle would be reduced to nothing (-10 + 10 = 0), and that is clearly not depicted in the graph.

This leaves only answer B as a reasonable possibility.
A company has a right cylindrical storage tank for storing natural gas. The tank measures 20 feet wide by 60 feet high.

The company estimates that it will cost $2.00 per square foot to paint the tank. Which of the following expressions could be used to determine the total cost to paint the top and the side of the tank?

A. \((1200\pi + 100\pi)(2.00)\)

B. \(\frac{(1200\pi + 100\pi)}{(2.00)}\)

C. \((1200\pi + 200\pi)(2.00)\)

D. \(\frac{1200\pi + 200\pi}{2.00}\)

The formula to calculate the surface area of a cylinder is as follows:

\[2\pi r^2 + 2\pi rh\]

A careful reading of the question indicates that we are trying to find out how much money it will cost to paint half of the cylinder and one of the ends – very important information. It means that we will only need half of the surface area formula to perform the calculation:

\[\pi r^2 + \pi rh\]
We drop the 2’s from the formula, because we’re only painting one cap and half the cylinder. The next task is to plug in the information:

\[
\pi r^2 + \pi rh
\]

\[
\pi (10^2) + \pi (20 \times 60)
\]

\[
100\pi + 1200\pi
\]

We now must account for the $2.00 per square foot. We place that in parentheses at the front or back of the equation:

\[
(100\pi + 1200\pi)(2.00)
\]

Since it looks much like item a, we select it.

Hopefully, you didn’t select items B or D. We can cross those off the list right away, because they involve division and this question requires multiplication. How do you know? *Because you’re not trying to find out how much of the cylinder you can paint if you have $100 dollars.* That would require division. In this case, you’re trying to figure out pure cost: I have this much area to paint and the paint costs $2.00 per square foot or x times 2.00.
have a basic understanding of linear equations and their properties (e.g., slope, perpendicularity); the multiplication, division, and factoring of polynomials; and graphing and solving quadratic equations through factoring and completing the square.

A competitor in an event at a hot-air balloon festival throws a marker down from an altitude of 200 feet toward a target. The table below shows the relationship between the height of the marker above the ground and the time since the competitor threw the marker.

<table>
<thead>
<tr>
<th>Time (sec.)</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1.0</th>
<th>1.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (ft.)</td>
<td>200</td>
<td>189</td>
<td>176</td>
<td>161</td>
<td>144</td>
<td>125</td>
</tr>
</tbody>
</table>

How many seconds after the competitor throws the marker will the marker hit the ground?
A. 2.0 seconds  
B. 2.5 seconds  
C. 3.0 seconds  
D. 3.5 seconds

The correct response is B.
This question is an easy one for you and your calculator.

In the first 0.25 seconds, the marker drops 11 feet. In the next 0.25 seconds, the marker drops 13 feet. And, in the next 0.25 seconds, the marker drops 15 feet. Continuing this pattern by extending the table and using your calculator:

<table>
<thead>
<tr>
<th>Time</th>
<th>0</th>
<th>.25</th>
<th>.5</th>
<th>.75</th>
<th>1.0</th>
<th>1.25</th>
<th>1.50</th>
<th>1.75</th>
<th>2.00</th>
<th>2.25</th>
<th>2.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>200</td>
<td>189</td>
<td>176</td>
<td>161</td>
<td>144</td>
<td>125</td>
<td>104</td>
<td>81</td>
<td>56</td>
<td>29</td>
<td>0</td>
</tr>
</tbody>
</table>

| Drop Rate | 0   | 11  | 13  | 15  | 17   | 19   | 21   | 23   | 25   | 27   | 29   |

The marker will hit the ground (height = 0) after 2.5 seconds
A deposit of $250.00 is made in an interest-paying savings account. The table shows the amount of money in the account at the end of each of three equal payment periods. No money is deposited or withdrawn from the account, and the interest rate remains constant.

<table>
<thead>
<tr>
<th>Deposit</th>
<th>$250.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>$254.38</td>
</tr>
<tr>
<td>Period 2</td>
<td>$258.83</td>
</tr>
<tr>
<td>Period 3</td>
<td>$263.36</td>
</tr>
</tbody>
</table>

Which of the following expressions could be used to determine the amount of money in the account after \( n \) payment periods?

A. \( 250 + (0.0175)^n \)
B. \( 250 + 250(0.0175)^n \)
C. \( 250 (0.0175)^n \)
D. \( 250(1.0175)^n \)

D is the correct response. Here is how to solve it.

First of all, start the known information.

- $250 dollars is the initial deposit
- The interest rate remains constant for three periods
- From the initial deposit of $250.00 to the first period, there is now $254.38 in the account or $4.38 of interest.

You now have enough information to answer the question.

First, write down the equation, where 250 times a particular rate equals $4.38.

\[
250r = 4.38
\]

Second, isolate the variable as with any algebraic equation. In this case, divide each side by 250:

\[
\frac{1}{250} 250r = \frac{4.38}{250} \quad \frac{1}{250}
\]

\[
r = 0.0175
\]
Third, write the equation for calculating the interest on $250. The new balance for each period will = $250 + $250 x .0175. This is the same as writing:

\[ 250 + (1.0175) \]

Fourth, write the equation next to each period in the table, noting the \( n \) for each period (in bold below):

<table>
<thead>
<tr>
<th>Deposit</th>
<th>$250.00</th>
<th>( 250 + (1.0175) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>$254.38</td>
<td>( 250(1.0175) )</td>
</tr>
<tr>
<td>Period 2</td>
<td>$258.83</td>
<td>( 250(1.0175) )</td>
</tr>
<tr>
<td>Period 3</td>
<td>$263.36</td>
<td>( 250(1.0175) )</td>
</tr>
</tbody>
</table>

The only answer that fits this pattern is D. \( 250(1.0175)^n \) where the \( n \) is an exponent that changes as the periods pass and the result of the equation will tell you the amount of money in the account after \( n \) payment periods.
Which of the following situations best illustrates the mathematical concept of a linear relationship—a relationship between two variables such that a change in one is accompanied by a proportional change in the other?

A. Joe's Pizza sells an 8-inch-diameter pizza for $10.00, a 12-inch-diameter pizza for $14.00, and a 16-inch-diameter pizza for $16.00.

B. College Painters estimates that two people could paint a house in ten days, four people could paint the same house in four days, and eight people could paint the house in one day.

C. A recycling center offers $22.00 for 100 pounds of scrap aluminum, $33.00 for 150 pounds, and $44.00 for 200 pounds.

D. The world's population doubled in the 15 years from 1960 to 1975, doubled again in the 20 years from 1975 to 1995, and is expected to double once more in the 30 years from 1995 to 2025.

Correct Response: C. The relationship in Response C is linear; the amount of money the recycling center will pay is directly proportional to the number of pounds of aluminum it receives.

This relationship may be demonstrated graphically as a straight line, as shown below.

As shown in the graph, doubling the amount of scrap aluminum will double the amount paid. Compared with the $22 the recycling center will pay for 100 pounds of scrap aluminum, it will pay 50 percent more ($33) for 50 percent more aluminum (150 pounds) and 100 percent more ($44) for 100 percent more aluminum (200 pounds). The price per 100 pounds remains constant, at $22 (which can also be expressed as 22 cents per pound).

Other Responses:
- Response A. This relationship is not linear. There is no linear relationship between the size of a pizza and its cost in this example.

- Response B. This relationship is not linear. The number of painters doubles from two to four and then doubles again from four to eight, but the time required to paint the house decreases from ten days to four days to one day.

- Response D. This relationship is not linear. The world's population doubled over a 15-year period, doubled again over a 20-year period, and is expected to double once more over a 30-year period.
A quadratic function \( h(x) \) has zeros at 4 and \(-3\) and a \( y \)-intercept of \(-12\). The function \( h(x) \) is translated \(-3\) units on the \( x \)-axis. Which of the following equations represents \( g(x) \), the transformation of \( h(x) \)?

A. \( g(x) = x^2 - 5x \)
B. \( g(x) = x^2 + 7x \)
C. \( g(x) = x^2 - x - 15 \)
D. \( g(x) = x^2 + 5x - 6 \)
In solving the quadratic equation

\[ x^2 + 14x - 4 = -30 \]

by completing the square, the first step is to add 4 to both sides of the equation. The second step is to:

A. add 49 to both sides of the equation.
B. factor \( x \) from the binomial \( x^2 + 14x \).
C. factor the number 2 from 14, \(-4\), and 30.
D. take the square root of both sides of the equation.

Completing the square is a trick to solving quadratic equations and is a part of pre-calculus; thus, you should know the name of the method and how it works, but don’t plan on having to apply the whole process to any questions on CSET Multiple Subjects.

This question already has the quadratic equation in the correct form for completing the square:

\[ x^2 + 14x - 4 = -30 \]

The question then tells you that the first step of adding 4 to each step of the question has already been done, and they are asking you what should happen next. To understand why adding 49 to each side is the correct response, let’s walk through the rest of the example. Adding 4 to each side of the equation results in the following:

\[ x^2 + 14x = -26 \]

In completing the square, the next step is to take 14, divide it by two, square it, and multiply the result on both sides of the equation. This step is a standard step in completing the square which explains why 49 is the correct response:

\[
\begin{align*}
\ x^2 + 14x & = -26 \\
14 ÷ 2 & = 7 \\
7^2 & = 49 \\
(49) + x^2 + 14x & = -26 + (49)
\end{align*}
\]
18. A student is using a computer program to graph the equation of a line in the form \( y = mx + b \). If both \( m \) and \( b \) are elements of the set \( \{1, 2, 3\} \), how many distinct lines can be drawn?

- A. 3
- B. 6
- C. 8
- **D. 9**

The graph of the equation \( 4y = 3x + 4b \), where \( b \) is a constant, has an \( x \)-intercept of 3. What is the value of \( b \)?

- A. \( \frac{16}{9} \)
- B. \( -2 \)
- C. \( \frac{9}{4} \)
- D. \( -4 \)
2H interpret graphs of linear and quadratic equations and inequalities, including solutions to systems of equations.

When the lines $y = x + 2$, $y = -4$, and $x = 6$ are graphed on the coordinate system below, what is the area of the polygon enclosed by the three lines?

A. 8 square units  
B. 32 square units  
C. 72 square units  
D. 144 square units

Correct Response: C. The first step is to graph the three lines on the given coordinate system. The polygon enclosed is a triangle with a base of 12 units and a height of 12 units. Since the area of a triangle is $\frac{1}{2}bh$, the area of this triangle is $\frac{1}{2}(12)(12) = 72$ square units.
The below graph represents a system of linear equations.

For what values of \( m \) will the solution to the system be in the first quadrant?

A. \( m < \frac{1}{2} \)

B. \( m < 5 \)

C. \( m > \frac{1}{2} \)

D. \( m > 5 \)

Correct Response: A. The graphic solution to a system of two linear equations is the point of intersection of the two lines. Since both lines are in the form \( y = mx + b \), one line has a slope of \( \frac{1}{2} \), and the slope of the second line, \( m \), must be determined. In order for the two lines to intersect, the value of \( m \) cannot equal \( \frac{1}{2} \). In order for the point of intersection to lie in the first quadrant, the value of \( m \) must be less than \( \frac{1}{2} \).
Which of the following is a correct graphical representation of the system below?

\[
\begin{align*}
2x + y &= 6 \\
y &= 3x + 1
\end{align*}
\]

A is the correct response. Of the four graphs, only choice A shows us the two equations correctly. Here is how to solve it.

**The First Equation: 2x + y = 6**
First, solve the first equation *twice* by solving for x and then y using zeros as place holders in the equation.

\[
\begin{align*}
2x + y &= 6 \\
2x + y &= 6 \\
2x + 0 &= 6 \\
2(0) + y &= 6 \\
2x &= 6 \\
y &= 6 \\
x &= 3 \\
(3,0)
\end{align*}
\]

Second, find the co-ordinates for the x and y equations. Remember that we order coordinates as (x, y). NOTE: One of the co-ordinates must be zero, because it was used to “zero” the equation to help us solve it.
\[2x + y = 6\]

<table>
<thead>
<tr>
<th>Solve for X</th>
<th>Solve for Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>[2x + y = 6]</td>
<td>[2x + y = 6]</td>
</tr>
<tr>
<td>[2x + 0 = 6]</td>
<td>[2(0) + y = 6]</td>
</tr>
<tr>
<td>[2x = 6]</td>
<td></td>
</tr>
<tr>
<td>[x = 3]</td>
<td>[y = 6]</td>
</tr>
<tr>
<td>(x, y)</td>
<td>(x, y)</td>
</tr>
<tr>
<td>(3,0)</td>
<td>(0,6)</td>
</tr>
</tbody>
</table>

Therefore, (3,0) is the x-intercept, and (0,6) is the y-intercept as shown in the table above. Plot the co-ordinates on the graph and then draw a line between them.

The Second Equation: \[y = 3x + 1\]
The second equation can be graphed more easily using the slope-intercept method, since this equation is already in the form

\[y = mx + b\]

where \(m\) is the slope and \(b\) is the y-intercept. In this case, the slope of the line is 3 and the y-intercept is 1, as shown below.

\[y = 3x + 1\]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
</tr>
</tbody>
</table>

The co-ordinates would show a line moving upward on the graph.
Superimposing the graphs of these two equations gives choice A, the correct response.

<table>
<thead>
<tr>
<th>2x + y = 6</th>
<th>y = 3x + 1</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph 1" /></td>
<td><img src="image2.png" alt="Graph 2" /></td>
<td><img src="image3.png" alt="Combined Graph" /></td>
</tr>
</tbody>
</table>

The slopes shown in choice B represent the reciprocals of the correct slopes. Although choice C represents a correct solution set, only one line has been correctly graphed. Choice D places the x-intercept on the y-axis and the y-intercept on the x-axis.
9. Use the graph below to answer the question that follows.

The graph represents an equation of the form $y = mx + b$. Which of the following statements about $m$ and $b$ are true?

A. $m > 0$ and $b > 0$
B. $m > 0$ and $b < 0$
C. $m < 0$ and $b > 0$
D. $m < 0$ and $b < 0$

10. Use the graph below to answer the question that follows.

Which of the following lines has the same slope as the line above and passes through the point (6, 12)?

A. $y - 2x = 0$
B. $y + 3x = 30$
C. $y - 3x = -6$
D. $y + 3x = 42$
Domain Three: Measurement and Geometry

3.1 Two- and Three-dimensional Geometric Objects

3A understand characteristics of common two- and three-dimensional figures, such as triangles (e.g., isosceles and right triangles), quadrilaterals, and spheres.

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>15. A circular area rug has a circumference of 26π inches.</td>
<td>B. 13 inches</td>
</tr>
<tr>
<td>What is the radius of the rug?</td>
<td></td>
</tr>
<tr>
<td>A. √26 inches</td>
<td></td>
</tr>
<tr>
<td>B. 13 inches</td>
<td></td>
</tr>
<tr>
<td>C. 26 inches</td>
<td></td>
</tr>
<tr>
<td>D. 39 inches</td>
<td></td>
</tr>
</tbody>
</table>

What is the measure of the angle θ between the two hands of an analog clock when the clock reads 5:00?

<table>
<thead>
<tr>
<th>Answer</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A. 120°</td>
<td></td>
</tr>
<tr>
<td>B. 150°</td>
<td></td>
</tr>
<tr>
<td>C. 160°</td>
<td></td>
</tr>
<tr>
<td>D. 175°</td>
<td></td>
</tr>
</tbody>
</table>
29. Use the diagram below to answer the question that follows.

A metal worker is building a wire frame (represented by the dotted and solid lines in the diagram above) for a cubic box that is to have a volume of 27 cubic feet. What is the total length of wire the metal worker will need?

A. 27 feet
B. 36 feet
C. 54 feet
D. 81 feet
Which of the following equations correctly expresses the relationship between the number of faces \((F)\), vertices \((V)\), and edges \((E)\) for the shapes in the table?

A. \(\frac{1}{4} F \times V = E\)
B. \(F + V = E + 2\)
C. \(F + V + E = 20\)
D. \(F + V + E = 26\)

The table gives the number of faces, vertices, and edges for several solid shapes. ANS A

<table>
<thead>
<tr>
<th>Shape</th>
<th>Faces</th>
<th>Vertices</th>
<th>Edges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>?</td>
<td>8</td>
<td>12</td>
<td>18</td>
</tr>
</tbody>
</table>
18. Which of the following shapes should be placed in the fourth row to complete the table? ANS D

A. 

B. 

C. 

D. 

3B draw conclusions based on the congruence, similarity, or lack thereof, of two figures.

If \( AB \) is parallel to \( CD \) and \( BC \) is parallel to \( DE \), what is the value of \( x - y \)?

A. \(-15\)
B. \(-1614\)
C. 15
D. 1712

A fifth-grade class is using pattern blocks in the shape of congruent equilateral triangles to devise and solve problems involving fractions. One group devises the problem illustrated below:

Given that the sum of Shapes A and B represents \( \frac{5}{8} \), which of the following represents \( 1 \frac{1}{4} \)?

A. 
B. 
C. 
D. 

ANS B
3C identify different forms of symmetry, translations, rotations, and reflections.

remember to add section from ADD SUBTEST
TWOMATH WORKBOOK QUESTIONS 1 31 05

8. Use the diagrams below to answer the question that follows.

If \( ABCDE \sim PQRST \), what is the ratio of the area of \( ABCDE \) to the area of \( PQRST \)?

A. 1:9
B. 1:4
C. 1:3
D. 1:2

Correct Response: B. Since \( ABCDE \) is similar to \( PQRST \), corresponding sides of the polygons are proportional. For the polygons in this problem, \( \frac{AB}{PQ} = \frac{BC}{QR} = \frac{1}{2} \). In other words, the length of each side of \( PQRST \) is twice the length of its corresponding side in \( ABCDE \). The area of \( ABCDE \) can be considered to be equal to the sum of \( N \) unit squares. The area of \( PQRST \) is therefore the sum of \( N \) squares, with each square having sides of length 2. Since the area of \( ABCDE \) is \( N \), the area of \( PQRST \) is \( 2 \times 2 \times N = 4N \). Hence, the area of \( ABCDE \) is \( \frac{N}{4N} = \frac{1}{4} \).

12. Use the diagrams below to answer the question that follows.

Correct Response: C. Since \( ABCDE \) is similar to \( PQRST \), corresponding sides of the polygons are proportional. For the polygons in this problem, \( AB \)
\( PQ = BC \)
\( QR = 12 \)

. In other words, the length of each side of \( PQRST \) is twice the length of its corresponding side in \( ABCDE \). The area of \( ABCDE \) can be considered to be equal to the sum of \( N \) unit squares. The area of \( PQRST \) is therefore the sum of \( N \) squares, with each square having sides of length 2. Since the area of \( ABCDE \) is \( N \), the area of \( PQRST \) is \( 2 \times 2 \times N = 4N \). Hence, the area of \( ABCDE \)
area of \( PQRST = N \)
\( 4N = 14 \)

. Choice A
If $ABCDE \sim PQRST$, what is the ratio of the area of $ABCDE$ to the area of $PQRST$?
A. 1:2
B. 1:3
C. 1:4
D. 1:9

is the ratio of the lengths of the corresponding sides of the polygons. Choice B is the ratio of $AB/AB + PQ$, and choice D is the square of this quantity.

24. Use the diagram below to answer the question that follows.

If the sequence above continues in the same pattern, how many small triangles would be needed to make the figure that would occur in Step 5?
A. 16
B. 25
C. 36
D. 49
13. Use the diagram below to answer the question that follows.

Which of the following statements describes how the value of \( y \) depends on the value of \( x \) in the triangles above?
A. \( y \) is directly proportional to \( x \).
B. \( y \) is directly proportional to the square of \( x \).
C. \( y \) is inversely proportional to \( x \).
D. \( y \) is inversely proportional to the square of \( x \).

Six of the squares in the diagram above have the areas indicated. What is the area of the seventh square?
A. 81
B. 100
C. 144
D. 169

Square corners, 5 units on a side, are removed from a 15-unitby-25-unit rectangular piece of cardboard. The sides of the cardboard are then folded to form an open box. Which of the following is the
volume, in cubic units, of the box?

A. 375  
B. 625  
C. 1000  
D. 1125  

An overhead light source projects parallelogram $ABCD$ to $A'B'C'D'$.  

Given that the projection is a dilation, which of the following expressions represents the perimeter of the smaller figure, $ABCD$, in terms of $x$?

A. $2x - 6$  
B. $3 + x/3$  
C. $3x - 18$  
D. $6 + 2/3x$

20. Use the diagram below to answer the question that follows.

A student places a flat mirror along each of the edges of the shape above, one edge at a time. Along which edge will the composite shape created by the image and the original shape form a quadrilateral?

A. RS  
B. ST
A pencil 5 inches long is held between a wall and a flashlight creating a shadow on the wall as shown above. What is the length of the pencil's shadow in inches?

A. 10  
B. 12  
C. 15  
D. 20

Which of the figures above has both reflective and rotational symmetry?

A. Figure A  
B. Figure B  
C. Figure C  
D. Figure D
Use the diagram below to answer the question that follows.

If the area of rectangle \( LMNO \) is 80, what is the area of the shaded rectangle?

A. 30  
B. 35  
C. 36  
D. 48

On your geoboard, construct 6 figures congruent to the figure shown below.

This activity is most useful for assessing a student's ability to:
A. calculate areas.  
B. construct similar figures.  
C. identify types of symmetry.  
D. apply translations, reflections, and rotations.
If the pattern continues, how many black boxes will there be in the fifth element of the pattern?

A. 16
B. 18
C. 20
D. **22**

3D understand the Pythagorean theorem and its converse.

2. The discovery of which of the following was a consequence of the theorem $a^2 + b^2 = c^2$ and is generally credited to the Greek school of the Pythagoreans?

A. the existence of irrational numbers
B. the congruence of base angles in isosceles triangles
C. the formula for the area of a triangle
D. the equation of a circle in the coordinate plane

**Correct Response:** A. Application of the theorem named after Pythagoras (circa 500 B.C.), the founder of the Pythagorean school, often leads to irrational numbers. For example, applying the theorem to find the length of a diagonal of a square with sides of length 1 results in $c^2 = 1^2 + 1^2$ or $c = 2$, which is irrational since it cannot be expressed in the form $\frac{a}{b}$, $b \neq 0$. There is strong historical evidence suggesting that the Pythagoreans were aware that numbers of this form could not be expressed as a ratio of two whole numbers, which caused the school much concern since it contradicted some of their fundamental philosophical beliefs.

6. Find the approximate length of the longest straight line that can be drawn on a 9-inch-by-11-inch sheet of paper.

A. 20 inches
B. 18 inches
C. 14.2 inches
D. 11 inches

28. Use the diagram below to answer the question that follows.
Correct Response: C. The first step is to draw a diagram.

Since the longest straight line that can be drawn in a rectangle is a diagonal, and that line divides the paper into two right triangles, applying the Pythagorean theorem $d^2 = a^2 + b^2$, where the diagonal is the hypotenuse, gives $d^2 = 11^2 + 9^2$. Therefore, $d = \sqrt{11^2 + 9^2} = \sqrt{121 + 81} = \sqrt{202} \approx 14.2$ inches.

The diagram above (not to scale) shows how an elementary student who is 4 feet tall estimates the height of a lamppost. The student stands 30 feet from the base of the lamppost and measures her shadow from the light as 6 feet long. Approximately how high is the lamppost?

A. 20 feet
B. **24 feet**
C. 36 feet
D. 45 feet

34. Use the diagram below to answer the question that follows.

The diagram shows the dimensions of a triangular field next to a school. To estimate the number of wildflowers growing in the field, students counted a total of 36 flowers in a randomly selected 3-feet-by-4-feet rectangular section. Assuming the section is a representative sample of the entire field, approximately how many flowers are in the entire field?

A. 17,500
B. 35,000
C. 90,000
D. 180,000
are able to work with properties of parallel lines.
### 3.2 Representational Systems, Including Concrete Models, Drawings, and Coordinate Geometry

3F use concrete representations, such as manipulatives, drawings, and coordinate geometry to represent geometric objects.

#### Question 13

ABCD is a rectangle in the coordinate plane. If the coordinates of point A are (-2, 1) and the coordinates of point C are (6, 3), which of the following are possible coordinates of points B and D?

A. (-3, 2) and (1, 6)
B. (3, -2) and (1, -6)
C. (-2, 3) and (6, 1)
D. (2, -3) and (-6, -1)

**Correct Response:** C. A figure formed by connecting the two given points and the two unknown points must be a rectangle, i.e., a quadrilateral in which the pairs of opposite sides are congruent and parallel, and whose adjacent sides intersect at right angles. Draw a set of coordinate axes and plot the two given points and the pairs of points in each response. Only the points in response C result in a figure meeting the specified criteria.

#### Question 2

An equilateral triangle is constructed on a set of coordinate axes. If one vertex is at the origin and another is at the point \((a, 0)\), which of the following would be the \(y\)-coordinate of the third vertex of the triangle?

A. \(\frac{\sqrt{3}a}{2}\)
B. \(\frac{3a}{4}\)
C. \(\frac{\sqrt{3}a}{4}\)
D. \(\frac{3a}{2}\)

**ANS A**

#### Question 3

The lines above represent a system of two linear equations. Which of the following equations could be solved to find the \(x\)-coordinate of point \(P\)?

A. \(-\frac{8}{6}x + 6 = \frac{1}{2}x + 2\)
B. \(-\frac{6}{2}x + 6 = 2x - 4\)
C. \(\frac{8}{2}x + 8 = \frac{1}{2}x + 2\)
D. \(\frac{6}{2}x + 6 = 2x - 4\)

**ANS A**
24. The vertices of a polygon drawn on an x-y coordinate system are given by the ordered pairs below. 
(1, 2), (2, 4), (5, 2), (4, 4) 
What type of polygon is described by these coordinates? 
A. kite  
B. rectangle  
C. rhombus  
D. trapezoid

A point is randomly selected within the rectangle shown in the diagram above. What is the probability that the y-coordinate of the point is less than or equal to 2? 
A. 18  
B. 29  
C. 14  
D. 12  

Use the diagram below to answer the question that follows.
Which of the following measurements associated with the triangles in the diagram above can be expressed as a nonterminating, nonrepeating decimal?

A. AD
B. AC
C. BC
D. DC

The diagrams above show several regular polygons with all possible diagonals drawn. Which of the following graphs best represents the relationship of the number of sides of a polygon to the number of diagonals?

ANS C

Use the diagram below to answer the question that follows.
A student places a compass point at C and draws a circle through point A. Which of the following statements best describes this circle?
A. the set of all points in the plane at a distance of 5 units from (2, –3)
B. the set of all points in the plane at a distance of 5 units from (–3, 2)
C. the set of all points in the plane at a distance of 4 units from (–3, 2)
D. the set of all points in the plane at a distance of 4 units from (–3, –2)

6. Eileen drove a total of 430 miles. She drove at 55 miles per hour for 2.5 hours and at 65 miles per hour for the rest of the trip. How many hours did Eileen drive at 65 miles per hour?
A. 4.5 hours
B. 6.3 hours
C. 7 hours
D. 8.7 hours

Objective 15: Apply the properties of linear equations and inequalities in one variable. Correct Response: A . Let \( t \) represent the amount of time Eileen drove at 65 miles per hour. Then, total distance = distance at 55 miles per hour + distance at 65 miles per hour. Using the formula distance = speed \( \times \) time, along with the numerical values given, results in
\[
430 = 55(2.5) + 65t, \text{ or } 430 = 137.5 + 65t
\]
This is a linear equation in one variable. To solve this equation for \( t \), first subtract 137.5 from both sides of the equation, and then divide both sides by the coefficient of \( t \), which is 65.
\[
430 - 137.5 = 65t \Rightarrow 292.5 = 65t \Rightarrow t = \frac{292.5}{65} = 4.5 \text{ hours}
\]
Choice B results from dividing 55 by 2.5 to determine the distance traveled at 55 miles per hour. Choice C results from incorrectly deriving the linear equation as \( 430 = 137.5 + 65(t - 2.5) \). Choice D results from incorrectly solving the original equation by adding 137.5 to one side and subtracting it from the other.
14. A farmer is planning to paint a barn. The dimensions of the barn and of all the doors and windows are known, rounded to the nearest meter. In which of the following units should the paint be ordered?
A. cubic meters  
B. square meters  
C. kilograms  
D. liters  
Correct Response: D. The dimensions of the barn are known in meters so the surface area to be painted can be calculated to the nearest square meter, but the paint must be ordered in units of volume. Liters is the most appropriate unit for measuring liquid quantities in the metric system.

Use the figure below to answer the question that follows.

Chains of regular polygons can be formed by placing individual polygons side by side. For example, the figure above shows a chain of four regular hexagons. Which of the following expressions represents the perimeter of a chain of n regular polygons if each polygon has s sides of length 1?
A. \((n + 1) (s)\)  
B. \((n + 2) (s – 2)\)  
C. \(n(s – 2) + 2\)  
D. \(n(s/2 + 2)\)  
Correct Response: C. All of the polygons, except the two end polygons, have two shared sides that do not affect the perimeter. Each of the two end polygons has one shared side that does not affect the perimeter. The expression \(n(s – 2)\) represents a perimeter that accounts for all of the polygons having two shared sides. To make up for the two unshared sides on the end polygons, 2 must be added to the expression. Therefore, \(n(s – 2) + 2\) represents the perimeter.

7. Use the diagram below to answer the question that follows. The diagram shows five different figures on a geoboard.

Which of the following statements about the figures is true?
A. All the figures have the same area.  
B. All the figures have the same perimeter.  
C. The sum of the internal angles of all the figures is the same.  
D. The corresponding sides of all the figures are
proportional.
A truncated cube is a polyhedron formed by cutting the corners off a cube.

ANS C
5. A class is using manipulative tiles in the shape of equilateral triangles to explore fractions. One of the students makes up the problem illustrated below.

If Shape 1 and Shape 2 equal $\frac{2}{3}$, what does Shape 1 + Shape 2 + Shape 3 equal?

What is the solution to this problem?
A. 5/6
B. 1
C. 11/3
D. 11/2

3.3 Techniques, Tools, and Formulas for Determining Measurements.

Max plans to paint the exterior walls of his garage. The garage has a pentagonal front and back and rectangular sides. The diagram shows the front of the garage. The sides are 8-foot-by-15-foot rectangles. One gallon of paint costs $18.50 and covers 350 square feet. Assuming he will need two coats of paint, how much will Max have to spend on paint?
A. $18.50
B. $37.00
C. $55.50
D. $74.00

Correct Response: C. Since each of the two sides of the garage is a rectangle with a length of 15 feet and a width of 8 feet, the area of each side is 120 square feet. The front and back of the garage are each a rectangle with a length of 10 feet and a width of 8 feet plus a triangle with a base of 10 feet and a height of 2 feet, so the areas of the front and back are 90 square feet each. The total area of the garage (not including the roof) is thus 420 square feet. Since two coats of paint are needed, the total area to be painted is 840 square feet. Because one gallon of paint covers 350 square feet and 840 = 2.4, three gallons of paint are needed, and the cost of three gallons of paint is 3($18.50) = $55.50.
A plant grows at a steady rate. The table above gives the height of the plant after several different weeks of growth. What is the height of the plant after 10 weeks?

A. 28 cm  
B. 30 cm  
C. 31 cm  
D. 33 cm

The diagram above shows a method for estimating $AB$, the width of a river. If $BC = 30$ m, $CD = 80$ m, and $BE = 60$ m, what is the width of the river?

A. 22.5 m  
B. 40 m  
C. 90 m  
D. 160 m

During a day, Jessie spent 20 minutes washing dishes, 2 hours 35 minutes mowing the lawn, and 13 hours fixing a faucet. How much total time did Jessie spend completing these tasks?

A. 3 hours 5 minutes  
B. 3 hours 20 minutes  
C. 4 hours 40 minutes  
D. 4 hours 55 minutes

3. Use the diagram below to answer the question that follows.
A piston is placed at the top of a cylinder filled with air. As shown in the diagram above, students place a total of three 2 kg bricks, one at a time, on top of the piston and measure the volume of air in the cylinder. Their data are given in the table below.

<table>
<thead>
<tr>
<th>Mass on Piston (kg)</th>
<th>Volume of Air in Cylinder (cubic centimeters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>280</td>
</tr>
<tr>
<td>4.0</td>
<td>140</td>
</tr>
<tr>
<td>6.0</td>
<td>93</td>
</tr>
</tbody>
</table>

What would be the approximate volume of the air in cubic centimeters if students placed two more 2 kg bricks on top of the piston?
A. 35  
B. 47  
C. 56  
D. 70  

Use the triangles below to answer the question that follows.
If \(\text{CAT}\) is similar to \(\text{DOG}\), what is the measure of \(m\) \(\text{DO}\)?
A. \(\frac{91}{3}\)
B. \(\frac{102}{3}\)
C. 24
D. 32

Use the diagram below to answer the question that follows.

Triangle \(\text{ABC}\) is similar to triangle \(\text{ADE}\). What is the length of segment \(\text{EC}\) if \(\text{AC} = 15\)?
A. 1.7
B. 5
C. 6
D. 11

When an irregularly shaped object of mass 22.0 g is dropped into the rectangular container shown above, the object sinks to the bottom and the water level rises 1.5 cm. What is the density of the object, rounded
to the nearest tenth?
A. 0.8 g/cm³  
B. 1.2 g/cm³  
C. 1.8 g/cm³  
D. 14.7 g/cm³

A right cylindrical container with a radius of 4 cm and a height of 6 cm has a leak in the bottom. The container is initially filled with water. After 1.5 hours the container is three-fourths full.

At what rate is water leaking from the container?

A. \( \frac{2}{45} \pi \text{cm}^3 / \text{min} \)  
B. \( \frac{4}{15} \pi \text{cm}^3 / \text{min} \)  
C. \( \frac{11}{15} \pi \text{cm}^3 / \text{min} \)  
D. \( \frac{8}{5} \pi \text{cm}^3 / \text{min} \)

The orbit of a satellite is approximately 250 miles above the surface of the earth, which has a diameter of approximately 8,000 miles. If the satellite travels a total of about 4,000,000 miles in space during a 240-hour period, it will orbit the earth approximately once every:

A. 90 minutes.  
B. 96 minutes.  
C. 180 minutes.  
D. 192 minutes.
The diagram shows a right trapezoidal prism. What is the area of a horizontal cross section of the prism, 3 units above and parallel to the base of the prism?

A. 42
B. 108
C. 126
D. 144

3K identify relationships between different measures within the metric or customary systems of measurements and estimate an equivalent measurement across the two systems.
3L calculate perimeters and areas of two-dimensional objects and surface areas and volumes of three-dimensional objects.

<table>
<thead>
<tr>
<th>Use the information below to answer the two questions that follow.</th>
<th>The perimeter of the outer track consists of two sides of the interior square that has a side length of 40 meters, plus two semicircles of radius 30 meters (half the length of the side of the interior square plus the width of the track). The perimeter of the two semicircles taken together is equivalent to the perimeter of a circle with a radius of 30 meters. The perimeter of a circle can be calculated as $2 \pi r$ where $r$ is the radius of the circle, so the total outer perimeter of the track can be represented as $2(40) + 2 \times (30)$, or approximately 268 meters. Of the four options offered, only option C is correct.</th>
</tr>
</thead>
<tbody>
<tr>
<td>outer edge</td>
<td>Option A results from calculating the outer perimeter using a circle with a radius of 20 meters. Option B from calculating the perimeter using a circle with a radius of 25 meters. Option D results from misinterpreting the diagram and adding 10 meters to the length of the straight section of the track.</td>
</tr>
<tr>
<td>A running track, represented in the above diagram by the shaded region, has a width of 10 meters. It surrounds a central area that consists of two semicircular regions attached to a square region.</td>
<td>What is the approximate perimeter of the outer edge of the track, in meters?</td>
</tr>
</tbody>
</table>
| | A. 206  
B. 237  
C. 268  
D. 288 |
| What is the approximate area of the track (shaded region) in square meters? | The area of the shaded region can be represented as the sum of the areas of two rectangles, i.e., $2(40 \text{ meters} \times 10 \text{ meters})$, and the area between two concentric circles. The area between the two concentric circles can be calculated as the difference between the area of a circle of radius 30 meters (half the length of the side of the interior square plus the width of the track) and the area of a circle of radius 20 meters (half the length of the side of the interior square). The formula for representing the area of a circle is $\pi r^2$ where $r$ is the radius of the circle. Thus the total area of the shaded region in the diagram is calculated as $2(40 \times 10) + (30)^2 - (20)^2$, which equals approximately 2,371 square meters. Of the four options offered, only option B is correct. |
| | Option A calculates the area without including the straight sections of the track. Option C uses the sum of the areas of the two circles instead |
| A. 1,571  
B. 2,371  
C. 4,884  
D. 7,083 |
of the difference between them, and option D uses the diameters instead of the radii of the circles in calculating the areas of the curved sections of the track.

11. Use the diagram below to answer the problem:

In \( \triangle ACE \), \( m \text{AD} \) and \( m \text{BE} \) are medians. If \( m \text{BE} = 12 \text{ cm} \), what is the length of \( m \text{PE} \)?

A. 6 cm  
B. 8 cm  
C. 9 cm  
D. 10 cm

Correct Response: B. The solution to this problem depends on the theorem in geometry that states that the medians of a triangle intersect at a point that is two-thirds of the distance from each vertex to the midpoint of the opposite side. Therefore, the length of \( m \text{PE} \) is two-thirds the length of \( m \text{BE} \).

\[
\frac{2}{3}(12) = 8
\]

Therefore, choice B is the correct response. Choices A, C, and D are results obtained when the geometric theorem has been recalled incorrectly.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>(0, 2, 2)</td>
</tr>
<tr>
<td>F</td>
<td>(6, 2, 2)</td>
</tr>
<tr>
<td>G</td>
<td>(6, 2, 0)</td>
</tr>
<tr>
<td>H</td>
<td>(6, 0, 0)</td>
</tr>
</tbody>
</table>

The \( x-y-z \)-coordinates of four of a rectangular solid's vertices are given in the table. What is the surface area of this solid?

A. 24  
B. 40  
C. 56  
D. 72

6. A rectangular floor that measures 78 inches by 102 inches is to be completely covered with square tiles. The tiles are available in sizes with only whole-number side lengths. What is the smallest number of uncut tiles that could be used to cover the floor?

A. 204  
B. 221  
C. 1,326  
D. 3,978
A hollow right circular cylinder with open ends has a radius of \( r \) and a height of \( 2r \). The cylinder is cut along one side and unrolled to form a rectangle. What are the dimensions of the rectangle? ANS B

A. \( \pi r \) and \( 2r \)  
B. \( 2\pi r \) and \( 2r \)  
C. \( \pi r \) and \( 2\pi r \)  
D. \( 2\pi r \) and \( 4\pi r \)

A cube has a surface area of 216 square inches. The dotted lines shown bisect the sides of the cube. Each side of the cube is sliced along the dotted lines to produce a number of smaller cubes. What is the sum of the surface areas of all the cubes produced?

A. 216 in.\(^2\)  
B. 432 in.\(^2\)  
C. 864 in.\(^2\)  
D. 2,592 in.\(^2\)

The area of a picture frame is 127 square inches. If 1 inch = 2.54 centimeters, what is the area of the frame in square centimeters?

A. 19.6850  
B. 50  
C. 322.58  
D. 819.3532

14. One acre is 4840 square yards. Which of the following could be the dimensions, in feet, of a one-acre rectangular
### Building Lot?

A. 40 feet × 363 feet  
**B. 120 feet × 363 feet**  
C. 120 feet × 484 feet  
D. 420 feet × 2000 feet

### 3M

Relate proportional reasoning to the construction of scale drawings or models.

### 3N

Use measures such as miles per hour to analyze and solve problems.

<table>
<thead>
<tr>
<th>If a bicycle's front wheel has a diameter of 27 inches, and the bicycle is traveling at 10 miles per hour, approximately how fast is the wheel turning in revolutions per minute (rpm)?</th>
<th>Correct Response: C. The speed of the bicycle is 10 miles hour. Converting this into inches per minute results in 10 miles hour × 5280 feet 1 mile × 12 inches 1 foot × 1 hour 60 minutes = 10,560 inches minute. The circumference of the front tire is equal to π multiplied by the diameter or 27π inches. Accordingly, 27π inches = 1 revolution. Therefore, 10,560 inches minute × 1 revolution 27π inches = 124 rpm (choice C). Choice A results from using the area of the circle instead of the circumference. Choice B results from treating the diameter as if it were the radius. Choice D results from treating the diameter as if it were the circumference.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. 18 rpm</td>
<td></td>
</tr>
<tr>
<td>B. 62 rpm</td>
<td></td>
</tr>
<tr>
<td>C. 124 rpm</td>
<td></td>
</tr>
<tr>
<td>D. 391 rpm</td>
<td></td>
</tr>
</tbody>
</table>

### On a Trip from El Paso to Texarkana

A car is moving along a highway at a uniform rate of speed. At 9:00 A.M. it is 14 of the way from El Paso to Texarkana. At 3:00 P.M. it is 7/10 of the way from El Paso to Texarkana. Approximately what fraction of the way from El Paso to Texarkana was the car at 1:00 P.M.?

| A. 11/20 | |
| B. 35/20 | |
| C. 13/20 | |
| D. 7/10 | |